Solution to Problem 101A

Find the pressure at the center of a once-molten Earth of radius, $R = 6.44 \times 10^6\ m$, and density, $\rho = 5600\ kg/m^3$. Since the acceleration due to gravity is linear with radius and $g(R) = 9.81\ m/s$, then

$$g(r) = \frac{g(R)}{R} r$$

For a fluid at rest:

$$\frac{dp}{dr} = -\rho g$$

Integrating:

$$\begin{aligned} p(r) = \int \frac{dp}{dr} dr &= \int -\rho \frac{g(R)}{R} r dr \\ &= -\frac{1}{2} \rho \frac{g(R)}{R} r^2 + C \end{aligned}$$

Evaluating the constant:

$$p(R) = p_A = -\rho \frac{g(R)}{2} R$$

$$\Rightarrow C = p_A + \rho \frac{g(R)}{2} R$$

So the pressure is given by:

$$p(r) = p_A + \frac{\rho R g(R)}{2} \left[1 - \left(\frac{r}{R} \right)^2 \right]$$

Then at the center of the molten Earth (neglecting p_A):

$$\begin{aligned} p(0) &= \frac{\rho R g(r)}{2} \\ &= \frac{(5600\ kg/m^3)(6.44 \times 10^6\ m)(9.81\ kg \cdot m/s^2)}{2} \\ &= 1.77 \times 10^{11} Pa \end{aligned}$$