Solution to Problem 101C:

Since the acceleration due to gravity is linear with radius and $g = g_0$ at $r = R$:

$$g(r) \quad = \quad g_0 r/R \tag{1}$$

For a fluid at rest:

$$\frac{dp}{dr} \quad = \quad -\rho g \quad = \quad -\frac{\rho g_0 r}{R} \tag{2}$$

Integrating

$$p(r) \quad = \quad \int \frac{dp}{dr}\, dr \quad = \quad \int -\, \rho \frac{g_0}{R}\, r\, dr \quad = \quad -\frac{\rho g_0}{2R}\, r^2 \; + \; C \tag{3}$$

where C is an integration constant and since $p = p_A$ at $r = R$ it follows that

$$C \quad = \quad p_A + \frac{\rho g_0 \; R}{2} \tag{4}$$

and the pressure in the interior is therefore given by

$$p(r) \quad = \quad p_A \; + \; \frac{\rho R g_0}{2} \left[1 - \frac{r^2}{R^2} \right] \tag{5}$$