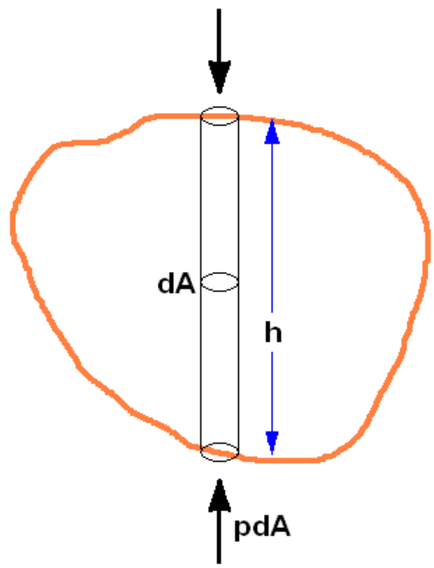
Solution to Problem 103A:

In any static body of fluid the pressure, p, varies with elevation, y, according to

$$\frac{\partial p}{\partial y} \quad = \quad -\rho g \tag{1}$$

where the density ρ and the acceleration due to gravity, g, may be functions y. Now consider the forces



acting on a body immersed in a fluid as shown above. Consider a vertical cylindrical element of cross-section, dA, and length, h, and denote the pressure in the fluid at the lower end by p. Then the pressure at the top end will be

$$p \;+\; \int_0^h \frac{\partial p}{\partial y}\, dy \quad = \quad p \quad - \int_0^h \rho(y)\; g\; dy \tag{2}$$

and therefore the net upward buoyancy force on the element dA will be

$$dA \int_0^h \rho(y)\; g\; dy \tag{3}$$

and the total buoyancy force will be given by integrating this over the whole of the displaced volume, V:

$$\int_V \left\{ \int_0^h \rho(y)\; g\; dy \right\}\; dA \tag{4}$$

But this expression is also just the weight of the displaced fluid and therefore Archimedes principle also holds for a compressible fluid.