Solution to Problem 105A

Find the particular value of the ratio b/h above which the configuration is stable.

Stability of the channel is determined by applying a slight rotation about the center of mass and then seeing whether it would right itself or fall over. The rotation shifts the center of buoyancy by further submerging one leg at the expense of the other. For the channel to be stable, the moment produced by the misalignment of the forces through the center of mass and the center of buoyancy must act counter to the small imposed rotation.

In the untilted configuration, the center of buoyancy is located at $\frac{h}{2}$ as shown in the diagram. The rotation shifts it to the left by $\frac{h}{2}\sin\theta$. If the COB moves to the right by more than this distance, a counter-clockwise torque will be applied and the channel will right itself.

We find the shift in the COB, l, by looking at the moments of the area of the submerged and emerged portions of the channel. These moments are related to the shift by:

$$l = \frac{\sum M_{CM}}{A}$$

where A is the total submerged area, which is the same before and after rotation (A=2ht). The moment of the area for each side is the area change multiplied by the distance from the center of mass:

$$\sum M_{CM} = \frac{bt}{2}\tan\theta\frac{b}{2\cos\theta} + \frac{bt}{2}\tan\theta\frac{b}{2\cos\theta}$$

Combining, simplifying, and considering small angle, θ, $(\tan\theta \approx \theta, \cos\theta \approx 1)$ leads to:

$$l = \frac{b^2\theta}{4h}$$

So for stability:

$$l > \frac{h}{2}\sin\theta \approx \frac{h}{2}\theta$$

$$\Rightarrow \frac{b}{h} > \sqrt{2}$$