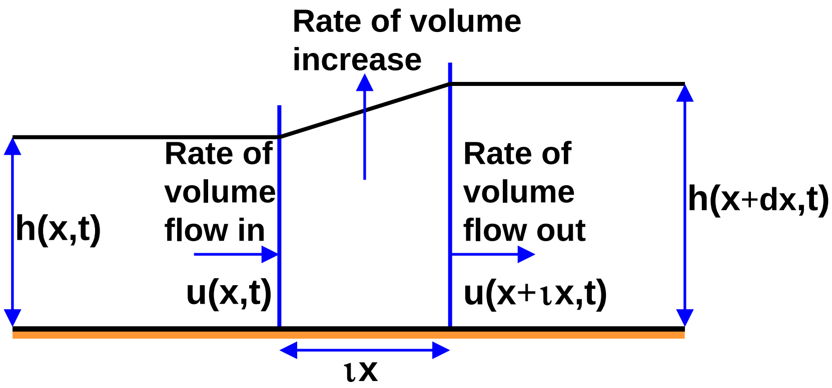
Solution to Problem 113A

To find an equation for the shallow water wave, a mass balance for the element δx will be used. The mass balance is given by

$$mass\ in = mass\ out$$

and three contributions can be identified:



- rate of volume increase per unit depth normal to the sketch: $\frac{\partial h}{\partial t}\delta x$
- rate of volume flow in per unit depth normal to the sketch: uh
- rate of volume flow out per unit depth normal to the sketch: $uh + \frac{\partial(uh)}{\partial x}\delta x$

The three contributions can be substituted into the mass conservation relation to yield

$$\begin{aligned} \frac{\partial h}{\partial t}\delta x + \left[uh + \frac{\partial(uh)}{\partial x}\delta x\right] - uh &= 0 \\ \frac{\partial h}{\partial t} + \frac{\partial(uh)}{\partial x} &= 0 \end{aligned}$$

This last differential equation is the first of two which comprise the shallow water wave equations that, with boundary conditions, must be solved to determine the two unknowns, $u(x)$, and $h(x)$. The second differential equation results from applying the momentum thereom to the same control volume.