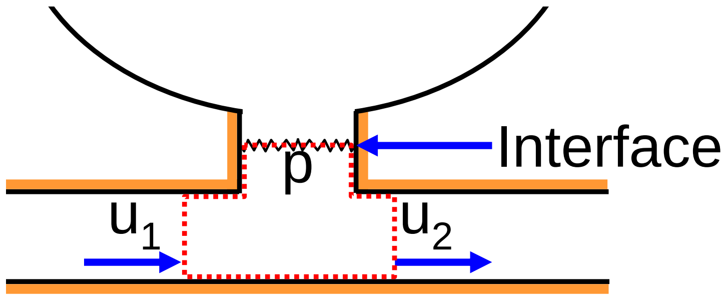
Solution to Problem 113B

Consider the control volume shown in red in which the upper surface is the moving liquid surface in the mouth of the accummulator:



Conservation of mass dictates that

$$\rho_L \frac{\mathrm{d}V_L}{\mathrm{d}t} + \rho_L A(u_2 - u_1) = 0$$

where V_L is the varying volume of liquid in the control volume and ρ_L is the liquid density.

The volume of gas in the accumulator, V, is related to the volume of liquid, V_L, in the control volume by $dV/dt = -dV_L/dt$. For the gas, $pV^\gamma = p_0 V_0^\gamma$, and therefore

$$\begin{aligned} \frac{\mathrm{d}V}{\mathrm{d}t} &= -\frac{V}{\gamma p}\frac{\mathrm{d}p}{\mathrm{d}t} \\ V &= V_0 \left(\frac{p_0}{p}\right)^{\frac{1}{\gamma}} \\ \therefore \;\; A(u_2 - u_1) &= -\frac{p_0^{\frac{1}{\gamma}} V_0}{\gamma} \frac{1}{p^{1+\frac{1}{\gamma}}} \frac{\mathrm{d}p}{\mathrm{d}t} \end{aligned}$$

Therefore the characteristic of the accumulator is

$$u_1 - u_2 = \frac{p_0^{\frac{1}{\gamma}} V_0}{\gamma A} \frac{1}{p^{1+\frac{1}{\gamma}}} \frac{\mathrm{d}p}{\mathrm{d}t}$$

Such a device is equivalent to a large capacitor to ground in an electrical circuit and is used to absorb fluctuations in the flowrate or pressure in a pipeline.