Solution to Problem 114A:

PART 1:

Streamlines: Streamlines are tangent to the velocity vectors at a particular moment in time, t. At this time the velocity components are

$$u_i = \frac{x_i}{(1+a_i t)} \tag{1}$$

and the streamline are therefore defined parametrically by $x_i(\eta)$ where η is a parametric variable and

$$\frac{dx_i}{d\eta} = \frac{x_i}{(1+a_i t)} \tag{2}$$

where it follows by integration that the streamlines at time t are given parametrically by

$$\frac{x_i(\eta)}{x_i(0)} = \exp\left(\frac{\eta}{1+a_i t}\right) \tag{3}$$

where $x_i(0)$ is a point on the streamline at $\eta = 0$.

Pathlines: Pathlines are routes traced out by individual particles. In this case the curve traced out by an individual particle is given by $x_i(t)$ where

$$u_i = \frac{dx_i}{dt} = \frac{x_i}{(1+a_i t)} \tag{4}$$

which by integration yields

$$\frac{x_i(t)}{x_i(0)} = [1 + a_1 t]^{1/a_i} \tag{5}$$

where $x_i = x_i(0)$ is a reference location for the streamline.

PART 2:

If $a_3 = 0$ the flows are planar in the (x_1, x_2) plane. With $a_1 = 2$ and $a_2 = 1$ streamlines are given by

$$x_1(\eta)/x_1(0) = \exp\left(\eta/1+2t\right) \quad ; \quad x_2(\eta)/x_2(0) = \exp\left(\eta/1+t\right) \tag{6}$$

and therefore the equation for the streamline is

$$x_1 = C x_2^{(1+t)/(1+2t)} \tag{7}$$

where C is some constant. In contrast the pathlines are described by

$$\frac{x_1(t)}{x_1(0)} = [1+2t]^{1/2} \quad ; \quad \frac{x_2(t)}{x_2(0)} = [1+t] \tag{8}$$

and therefore the equation for the pathlines is

$$\left\{\frac{x_1}{x_1(0)}\right\}^2 = 2\left\{\frac{x_2}{x_2(0)}\right\} - 1 \tag{9}$$

PART 3:

If all $a_i = 1$ the streamlines are given by

$$\frac{x_i(\eta)}{x_i(0)} = \exp\left(\frac{\eta}{1+t}\right) \tag{10}$$

and are therefore straight lines.

The pathlines are given by

$$\frac{x_i(t)}{x_i(0)} = [1+t] \tag{11}$$

and these are also straight lines.