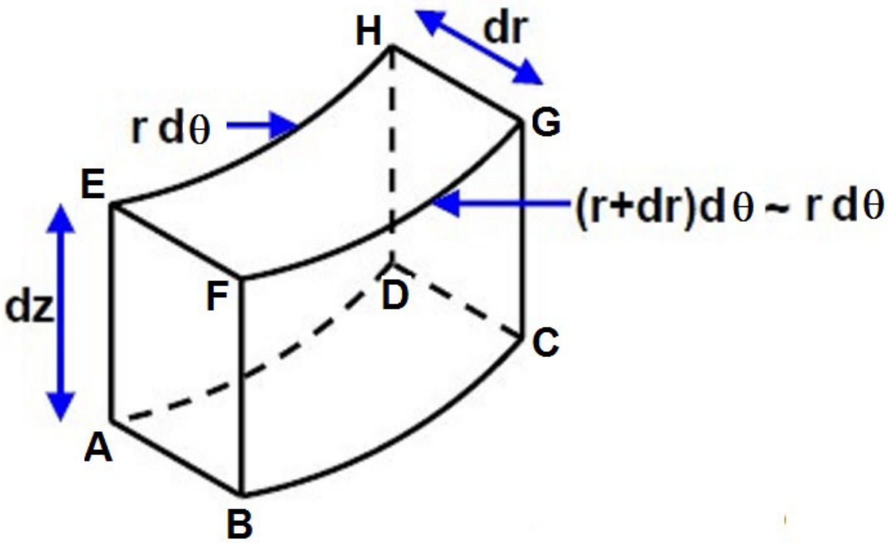
Solution to Problem 114C:

Consider the elemental control volume in cylindrical coordinates (r, θ, z) as sketched below:



The mass flux of the fluid of density ρ into the control volume through the side AEHD is

$$\rho\ u_r\ r\ d\theta\ dz \tag{1}$$

and the mass flux out through BFGC is

$$\rho\ u_r\ r\ d\theta\ dz\ + \frac{\partial(\rho u_r)}{\partial r} dr\ r\ d\theta\ dz \tag{2}$$

where we have neglected terms second order in dr and $d\theta$. Therefore the net flux in through sides AEHD and BFGC is

$$\frac{\partial \rho u_r}{\partial r}\ dr\ rd\theta\ dz \tag{3}$$

Similarly the net flux in through sides ABFE and DCGH is

$$\frac{1}{r}\frac{\partial \rho u_\theta}{\partial \theta}\ dr\ rd\theta\ dz \tag{4}$$

and the net flux in through sides ABCD and EFGH is

$$\frac{\partial \rho u_z}{\partial z}\ dr\ rd\theta\ dz \tag{5}$$

The sum of these net mass fluxes in must therefore be equal to the increase in mass within the control volume which is given by

$$\frac{\partial \rho}{\partial t}\ dr\ rd\theta\ dz \tag{6}$$

Therefore the continuity equation in this cylindrical coordinate system is

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r u_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial \theta} + \frac{\partial(\rho u_z)}{\partial z} \quad = \quad 0 \tag{7}$$

where (u_r, u_θ, u_z) denote the velocity components in the (r, θ, z) directions.