Solution to Problem 115H

The streamfunction for this planar incompressible flow is given as

$$\psi = A(x^2 y - y^3/3)$$

where A is a known constant.

a) It follows that the velocity components are

$$u = \frac{\partial \psi}{\partial y} = A(x^2 - y^2)$$

$$v = -\frac{\partial \psi}{\partial x} = -2Axy$$

b) By definition the vorticity, ω, is given by:

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

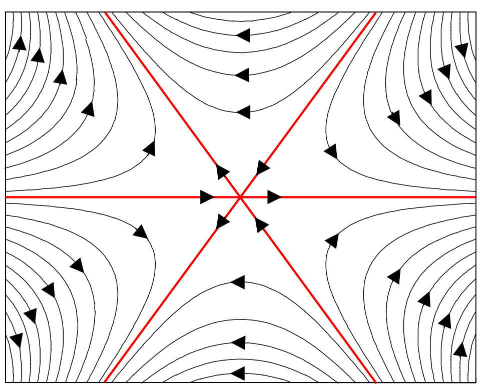
and therefore the flow is irrotational.

c) Construct the streamline for $\psi = 0$:

$$y(x^2 - y^2/3) = 0$$

so $\psi = 0$ on the lines $y = 0$ and $y = \pm\sqrt{3}x$.

Also note that on $y = 0$ we have $u = Ax^2$ and $v = 0$. And along $x = 0$ we have $u = -Ay^2$ and $v = 0$. Thus the flow is:



d) Since the flow is irrotational, inviscid and incompressible so we can use Bernoulli's equation to determine the pressure:

$$p + \frac{1}{2}\rho|u|^2 = const$$

$$|u|^2 = u^2 + v^2 = A^2(x^2 + y^2)^2$$

$$\therefore\; p + \frac{1}{2}\rho A^2(x^2 + y^2)^2 = const$$

Setting $p = p_0$ at the origin this yields

$$p = p_0 - \frac{1}{2}\rho A^2(x^2 + y^2)^2$$

A line of constant pressure (known as an isobar) is therefore a circle centered at the origin.