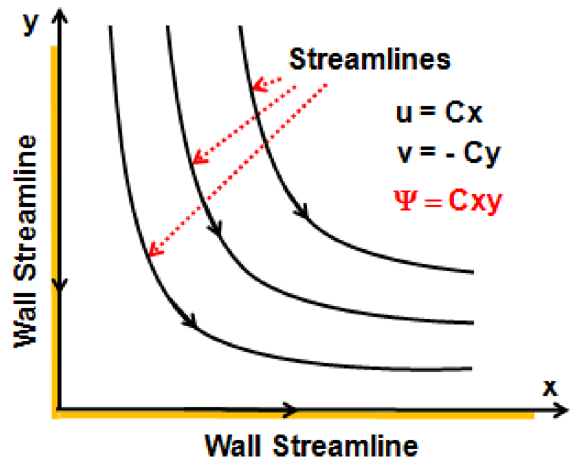
Solution to Problem 115J:

The following simple flows:

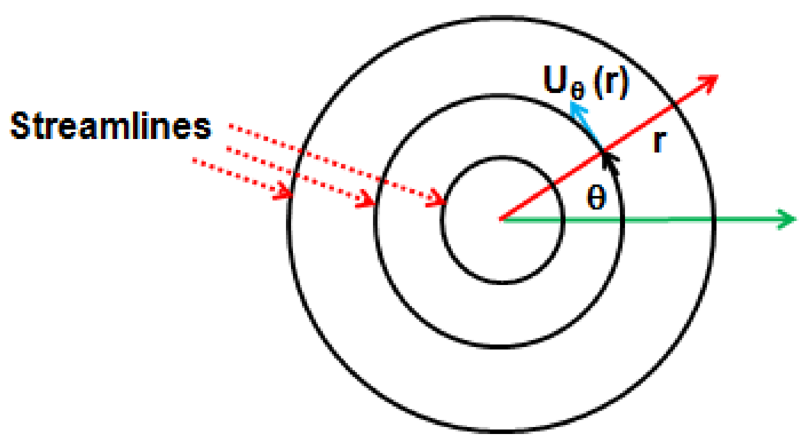
[A] Stagnation Point Flow, $(u, v, w) = (kx, -ky, 0)$: The streamlines are rectangular hyperbolae in which



$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \quad (1)$$

so this flow is irrotational.

[B] Free Vortex Flow, $(u_r, u_\theta, u_z) = (0, -\Gamma/2\pi r, 0)$: The streamlines are circles with



$$u = -u_\theta \sin\theta = -\frac{\Gamma}{2\pi}\sin\theta = \frac{\Gamma}{2\pi r}\frac{y}{(x^2+y^2)} \quad (2)$$

$$v = u_\theta \cos\theta = -\frac{\Gamma}{2\pi}\cos\theta = -\frac{\Gamma}{2\pi}\frac{x}{(x^2+y^2)} \quad (3)$$

$$w \;=\; 0 \tag{4}$$

and

$$\omega \;=\; \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \;=\; 0 \tag{5}$$

so this flow is irrotational.

[C] Solid Body Rotation: $(u_r, u_\theta, u_z) = (0, \Omega r, 0)$:

$$u \;=\; -\Omega r \sin\theta \;=\; -\Omega y \tag{6}$$

$$v \;=\; \Omega r \cos\theta \;=\; \Omega x \tag{7}$$

$$w \;=\; \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \;=\; \Omega - (-\Omega) \;=\; 2\Omega \tag{8}$$

Note the vorticity is uniform everywhere and equal to twice the rate of rotation. So this flow is not irrotational.