Solution to Problem 116A:

In this flow within a rotating cylinder containing compressible fluid:

$$u_z \;=\; 0 \;;\; \frac{\partial}{\partial t} \equiv \;0 \;\;;\; \frac{\partial}{\partial z} \equiv \;0 \;;\; u_r \;=\; 0 \;;\; u_\theta \;=\; \Omega r \;;\; f_r \;=\; f_\theta \;=\; f_z \;=\; 0 \tag{1}$$

The equation of motion in the z direction yields $\partial p \, \partial z = 0$ which is already established.

The equation of motion in the θ direction yields $\partial p \, \partial \theta = 0$ and hence as one would expect, the pressure, p, is a function only of the radial position, r.

The equation of motion in the r direction yields

$$\rho \left[\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + u_z \frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r} \right] \;=\; -\frac{\partial p}{\partial r} \tag{2}$$

and since the first four terms in the square brackets are all zero and the fifth term is equal to $\Omega^2 r$ it follows since that the pressure p is a function only of r;

$$\frac{\partial p}{\partial r} \;=\; \frac{dp}{dr} \;=\; \rho \Omega^2 r \tag{3}$$

Since $p = A\rho^k$ it follows that

$$dp \;=\; \left\{ \frac{p}{A} \right\}^{1/k} \Omega^2 r dr \tag{4}$$

and by integration

$$p^{(k-1)/k} \;=\; (k-1)\Omega^2 r^2 / 2kA^{1/k} \;+\; \text{constant} \tag{5}$$

and since $p = p_0$ at $r = 0$

$$p^{(k-1)/k} \;=\; p_0^{(k-1)/k} \;+\; (k-1)\Omega^2 r^2 / 2kA^{1/k} \tag{6}$$