Solution to Problem 116D

As stated this flow is

- Unsteady flow
- Incompressible, inviscid flow
- $u = U(t), v = w = 0$
- negligible body forces

Use these in Euler's equations:

$$\frac{D\vec{u}}{Dt} = -\frac{1}{\rho}\nabla p + \vec{f}$$

With v and w equal to zero and neglecting body forces, these reduce to:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial p}{\partial y} = 0, \qquad \frac{\partial p}{\partial z} = 0$$

The last two expressions imply that $p = p(x,t)$. Since $u = U(t)$, the equation in the x direction becomes:

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

Integrating:

$$\int \partial p = \int -\rho\frac{dU}{dt}dx = -\rho\frac{dU}{dt}\int dx$$

since ρ and $\frac{dU}{dt}$ are independent of x. Therefore

$$p(x,t) = -\rho\frac{dU}{dt}x + f(t)$$

and, eliminating $f(t)$ using the values at the two ends yields the result

$$p_2 - p_1 = -\rho\frac{dU}{dt}L$$

Note that if you visualize the fluid in the pipe as a Lagrangian mass of mass ρLA where A is the cross-sectional area and recognize that the net force acting on this mass in the positive x direction is $(p_1 - p_2)A$ then the above result is simply Newton's law of motion for that mass.