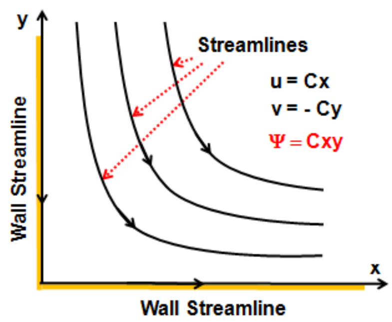
Solution to Problem 116F:



Since $\psi = Axy$ it follows that

$$u = \frac{\partial \psi}{\partial y} = Ax \; ; \; v = -\frac{\partial \psi}{\partial x} = -Ay \tag{1}$$

and

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0 \tag{2}$$

[A] Since the vorticity is zero, the flow is irrotational.

[B] Bernoulli's equation applies since the flow is steady, inviscid, incompressible and irrotational. Therefore

$$p + \frac{1}{2}\rho\,|\underline{u}|^2 + \rho g y = \text{constant} \tag{3}$$

since y is vertically upward. But

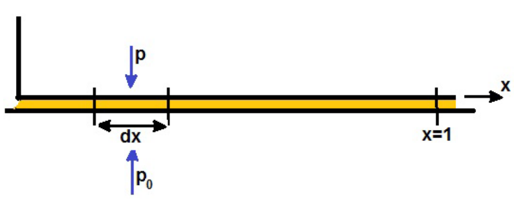
$$|\underline{u}|^2 = u^2 + v^2 = A^2(x^2+y^2) \tag{4}$$

Therefore

$$p = \text{constant} - \frac{1}{2}\rho A^2(x^2+y^2) - \rho g y \tag{5}$$

and since $p = p_0$ at $x = y = 0$:

$$p = p_0 - \frac{1}{2}\rho A^2(x^2+y^2) - \rho g y \tag{6}$$



[C] The net upward force per unit depth on an element dx of the plate at $y = 0$ is $(p_0 - p)dx$ as shown above. Therefore the upward force per unit depth on the wall between $x = 0$ and $x = 1$ is

$$= \int_0^1 (p_0 - p)dx \;=\; \int_0^1 \frac{1}{2}\rho A^2 x^2 dx \;=\; \frac{\rho A^2}{6} \tag{7}$$