Solution to Problem 117B:

[A] Beginning with the streamfunction, ψ:

$$\psi \;=\; Ur(1-r_0^2/r^2)\sin\theta \tag{1}$$

we note that the velocity components are

$$u_r \;=\; \frac{1}{r}\frac{\partial\psi}{\partial\theta} \;= U(1-r_0^2/r^2)\cos\theta \quad ; \quad u_\theta \;=\; -\frac{\partial\psi}{\partial r} \;=\; -U(1+r_0^2/r^2)\sin\theta \tag{2}$$

and therefore the vorticity, ω, is

$$\omega(r,\theta) \;=\; \frac{1}{r}\frac{\partial(ru_\theta)}{\partial r} - \frac{1}{r}\frac{\partial u_r}{\partial\theta} \;=\; 0 \tag{3}$$

and therefore the flow is irrotational.

[B] Calculating $e_{xy}(r,\theta)$:

$$e_{xy}(r,\theta) \;=\; \frac{1}{2}\left[\frac{\partial u}{\partial y} \;+\; \frac{\partial v}{\partial x}\right] \;=\; \frac{1}{2}\left[\frac{\partial^2\psi}{\partial y^2} \;-\; \frac{\partial^2\psi}{\partial x^2}\right] \tag{4}$$

and if we write ψ as a function of x and y:

$$\psi \;=\; Uy\left[1-\frac{r_0^2}{(x^2+y^2)}\right] \tag{5}$$

then

$$\frac{\partial\psi^2}{\partial y^2} \;=\; \frac{2Ur_0^2y(3x^2-y^2)}{r^6} \quad \text{and} \quad \frac{\partial\psi^2}{\partial x^2} \;=\; -\frac{2Ur_0^2y(3x^2-y^2)}{r^6} \tag{6}$$

and

$$e_{xy} \;=\; \frac{4Ur_0^2y(3x^2-y^2)}{r^6} \;=\; \frac{4Ur_0^2(1+2\cos 2\theta)\sin\theta}{r^3} \tag{7}$$

[C] Since the flow is irrotational, Bernoulli's equation applies and

$$\frac{p}{\rho} \;+\; \frac{1}{2}(u_r^2+u_\theta^2) \;=\; \frac{p_\infty}{\rho} \tag{8}$$

and therefore

$$\frac{2(p-p_\infty)}{\rho U^2} \;=\; 1+(r_0/r)^4-2(r_0/r)^2\cos 2\theta \tag{9}$$