Solution to Problem 120A

The stream function for the corner flow, ψ_c, is given as

$$\psi_c = Axy$$

The stream function and velocity potential of a given potential flow are related by

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}, \qquad v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}$$

Using this relationship to calculate the velocity potential of the corner flow from the known stream function yields

$$\frac{\partial \psi_c}{\partial x} = Ay = -\frac{\partial \phi_c}{\partial y}$$

$$\phi_c = -\frac{1}{2}Ay^2 + f(x)$$

and

$$\frac{\partial \psi_c}{\partial y} = Ax = \frac{\partial \phi_c}{\partial x}$$

$$\phi_c = \frac{1}{2}Ax^2 + g(y)$$

Combining the two equations yields

$$\phi_c = \frac{1}{2}A\left(x^2 - y^2\right)$$

where the constant that arises is taken to be zero.

The velocity potential for a sink, is defined as

$$\phi_s = k \ln r$$

where the strength k of the sink is to be determined. The volume flow rate per unit depth removed from the flow is given as q which can be related to k by integrating over the corner for which the sink acts

$$-q = \int_0^{\frac{\pi}{2}} u_r r d\theta$$

where

$$u_r = \frac{\partial \phi}{\partial r} = \frac{k}{r}$$

Thus,

$$k = -\frac{2q}{\pi}$$

Substituting k into the expression for the velocity potential of the sink yields

$$\phi_s = -\frac{2q}{\pi}\ln r = -\frac{q}{\pi}\ln r^2 = -\frac{q}{\pi}\ln\left(x^2 + y^2\right)$$

The velocity for the entire flow is the velocity potential of the corner flow, ϕ_c, plus the velocity potential of the sink, ϕ_s,

$$\phi = \underbrace{\frac{1}{2}A\left(x^2 - y^2\right)}_{\text{Corner Flow}} \underbrace{-\frac{q}{\pi}\ln\left(x^2 + y^2\right)}_{\text{Sink}}$$

The string will be pushed toward the origin as long as the velocity vector of the flow along the wall points toward the origin (i.e. $u|_{y=0} < 0$). The velocity of the flow in the x-direction is

$$u = \frac{\partial \phi}{\partial x} = Ax - \frac{2qx}{\pi\left(x^2 + y^2\right)}$$

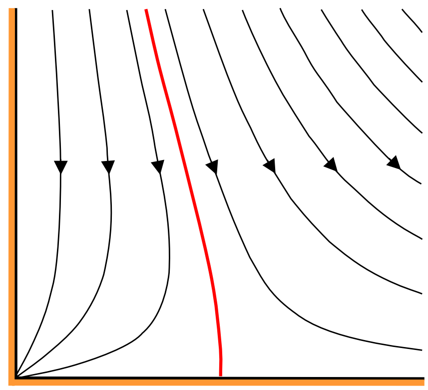
Evaluating this expression along the wall gives

$$u|_{y=0} = Ax - \frac{2q}{\pi x}$$

Solving for when the velocity changes sign ($u_{y=0} = 0$)

$$\begin{aligned} Ax &= \frac{2q}{\pi x} \\ x^2 &= \frac{2q}{\pi A} \\ \therefore \quad x &= \left(\frac{2q}{\pi A}\right)^{\frac{1}{2}} \end{aligned}$$

Since the string is positioned at $x = H$, the string will extend toward the origin if



$$H < \left(\frac{2q}{\pi A}\right)^{\frac{1}{2}}$$