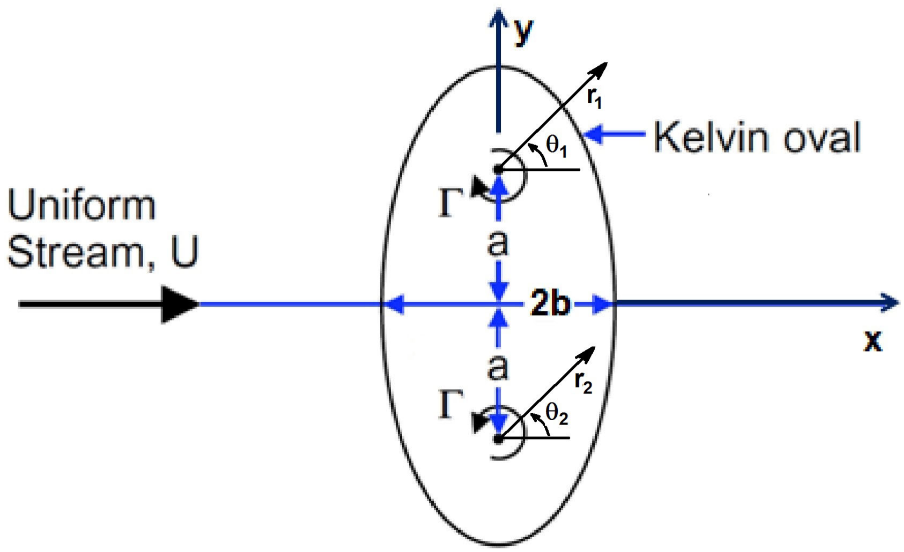
Solution to Problem 120E:

To evaluate the length of the Kelvin oval, we first compute the location of the front and rear stagnation points on the x axis. The potential flow is generated by the superposition of a uniform stream and two



potential vortices as follows:

$$\phi = Ux - \frac{\Gamma}{2\pi}\theta_1 + \frac{\Gamma}{2\pi}\theta_2 \quad (1)$$

$$\psi = Uy + \frac{\Gamma}{2\pi}\ln r_1 - \frac{\Gamma}{2\pi}\ln r_2 \quad (2)$$

where

$$r_1 = [x^2 + (y-a)^2]^{1/2} \quad \text{and} \quad \theta_1 = \arctan{(y-a)/x} \quad (3)$$

$$r_2 = [x^2 + (y+a)^2]^{1/2} \quad \text{and} \quad \theta_2 = \arctan{(y+a)/x} \quad (4)$$

Now to find the velocity in the x direction:

$$u = \frac{\partial\phi}{\partial x} = \frac{\partial\psi}{\partial y} \quad (5)$$

$$u = U + \frac{\Gamma}{4\pi}\left[\frac{2(y-a)}{x^2+(y-a)^2} - \frac{2(y+a)}{x^2+(y+a)^2}\right] \quad (6)$$

and on the x axis:

$$u_{y=0} = U - \frac{\Gamma a}{\pi(x^2+a^2)} \quad (7)$$

Finding the points on the x axis at which $u = 0$ to obtain the front and rear stagnation points:

$$L = 2a\left[\frac{\Gamma}{\pi a U} - 1\right]^{1/2} \quad (8)$$