

Problem 120J OR K

The velocity potential for the total flow is given by

$$\phi = Ux + \underbrace{\frac{B(x+a)}{(x+a)^2 + y^2}}_{\text{Uniform stream}} + \underbrace{\frac{B(x-a)}{(x-a)^2 + y^2}}_{\text{Doublet } @ x = -a} - \underbrace{\frac{By}{(x+a)^2 + y^2}}_{\text{Doublet } @ x = a} - \frac{By}{(x-a)^2 + y^2}$$

The velocities are given by

$$u = \frac{\partial \phi}{\partial x} = U + \frac{B \{ y^2 - (x+a)^2 \}}{ \{ (x+a)^2 + y^2 \}^2 } + \frac{B \{ y^2 - (x-a)^2 \}}{ \{ (x-a)^2 + y^2 \}^2 }$$
$$v = \frac{\partial \phi}{\partial y} = - \frac{2B(x+a)y}{ \{ (x+a)^2 + y^2 \}^2 } - \frac{2B(x-a)y}{ \{ (x-a)^2 + y^2 \}^2 }$$

At the critical value for B, the bodies touch at the origin, which means that the origin should be a stagnation point, with $u=0$.

$$u|_{x=0,y=0} = U - \frac{2B}{a^2}$$

Therefore the condition for flow around a single body is

$$B > \frac{Ua^2}{2}$$