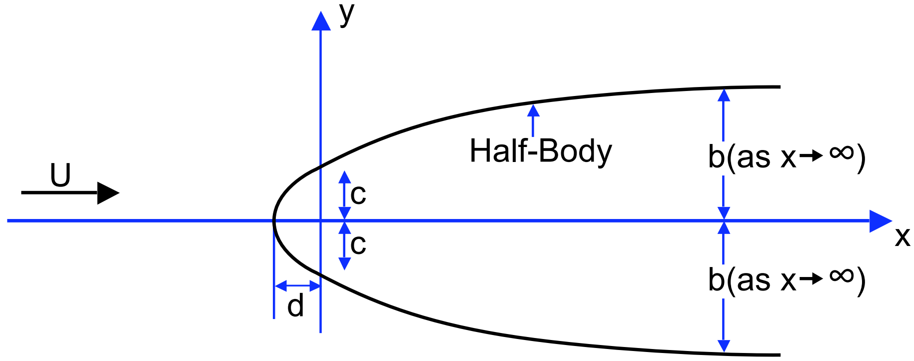
Solution to Problem 120N:

The planar potential flow of an incompressible, inviscid fluid past a Rankine half-body is formed by superposition of a source and a uniform stream: so that the velocity potential, ϕ, the streamfunction, ψ,



and the velocities, u_r and u_θ, are given by

$$\phi \;=\; Ux + \frac{Q}{4\pi}\ln\left(x^2+y^2\right) \;=\; Ur\cos\theta + \frac{Q}{2\pi}\ln r \tag{1}$$

$$u_r \;=\; U\cos\theta + \frac{Q}{2\pi r} \quad ; \quad u_\theta \;=\; U\sin\theta \tag{2}$$

$$\psi \;=\; Ur\sin\theta + \frac{Q\theta}{2\pi} \tag{3}$$

where $x = r\cos\theta$ and $y = r\sin\theta$.

The streamline that defines a Rankine half-body crosses the x axis (which is also a streamline) at the front stagnation point. The distance between the front stagnation point and the origin, d, is obtained by noting that the velocity, u_r, on the negative x axis is given by

$$(u_r)_{\theta=0} \;=\; -U + \frac{Q}{2\pi r} \tag{4}$$

and therefore $(u_r)_{\theta=0}$ is zero when $r = Q/2\pi U$. Therefore $d = Q/2\pi U$.

Next we note that the value of the streamfunction on the negative x axis is $\psi = Q/2$ and this must also be the value of the streamfunction on the Rankine half-body streamline surface. Therefore the shape of the Rankine half-body is given by the equation

$$(\psi)_{\text{Rankine halfbody}} \;=\; \frac{Q}{2} \;=\; Ur\sin\theta + \frac{Q\theta}{2\pi} \tag{5}$$

On the y axis, $\theta = \pi/2$, this yields $y = Q/4U$ so the distance c indicated in the figure is $c = Q/4U$. Finally the above relations show that far downstream as $x \to \infty$ the y coordinate of the half-body must asymptote to $Q/2U$ and therefore b, the half-width of the body far downstream, must be $Q/2U$. These geometric evaluations demonstrate that there is a family of shapes of Rankine half-bodies that become increasingly streamlined as the dimension Q/U becomes smaller.