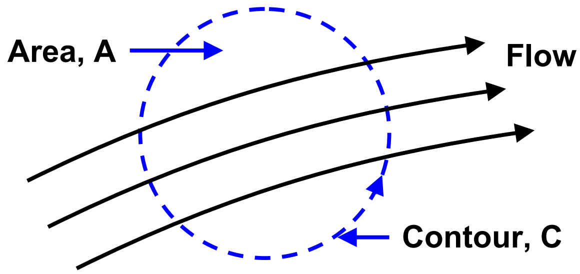
Solution to Problem 140A:

Consider a closed contour, C, enclosing a surface, A, in the planar flow of an incompressible fluid (the area, A, contains only fluid): The coordinate s is measured along the contour C and the "circulation", Γ,



is defined as the line integral of the fluid velocity, $\underline{u}$, around the contour C:

$$\Gamma = \int_C \underline{u}.\underline{ds}$$

Then, by Stokes' theorem

$$\Gamma = \int_A (\nabla \times \underline{u}).\underline{n}\; dA = \int_A \underline{\omega}.\underline{n}\; dA$$

where $\underline{n}$ is the unit vector normal to the surface A and $\underline{\omega}$ is the vorticity. For planar flow

$$\Gamma = \int_A \omega\; dA$$

In words, the circulation around C is equal to the total amount of vorticity inside A.

Moreover, if ω is zero inside A (the flow is irrotational) then clearly $\Gamma = 0$.