Solution to Problem 150O:

The Navier-Stokes equations under the conditions given reduce to a single equation of motion in the direction parallel to the plate:

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

for the velocity, $u(y,t)$, parallel to the plate where y is the distance normal to the plate.

We will seek a solution by separation of variables in which $u = Y(y)T(t)$. Then

$$Y \frac{dT}{dt} = \nu T \frac{d^2Y}{dy^2} \quad \text{or} \quad \frac{1}{T}\frac{dT}{dt} = \frac{\nu}{Y}\frac{d^2Y}{dy^2} = \lambda = \text{constant} \tag{2}$$

Therefore

$$T = Ae^{\lambda t} \quad \text{and} \quad Y = Be^{y\sqrt{\lambda/\nu}} + Ce^{-y\sqrt{\lambda/\nu}} \tag{3}$$

where A, B and C are constants. But B must be zero for the velocity far some the plate to tend to zero and therefore

$$u = De^{\lambda t}\, e^{-y\sqrt{\lambda/\nu}} \quad \text{and} \quad u_{y=0} = De^{\lambda t} \tag{4}$$

But by the no slip condition $u_{y=0} = Ue^{kt}$ and therefore D=U and $\lambda = k$ therefore

$$u = Ue^{kt}\, e^{-y\sqrt{k/\nu}} \tag{5}$$

The vorticity becomes

$$\omega = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \left(\frac{kU^2}{\nu}\right)^{1/2} e^{kt}\, e^{-y\sqrt{k/\nu}} \tag{6}$$

The distance $y = \delta$ at which the velocity has declined to $0.1U$ is given by

$$e^{-\delta\sqrt{k/\nu}} = 0.1 \quad \text{or} \quad \delta = 2.30\sqrt{\nu/k} \tag{7}$$