Solution to Problem 160D

Consider the volume flow rate for a stage with n tubes:

$$Q = nA_n\overline{u}_n$$

From the solution for laminar Poiseuille flow it follows that:

$$\overline{u} = \frac{R^2}{8\mu}\left(\frac{\partial p}{\partial x}\right)$$

and therefore,

$$A_1\frac{R_1^2}{8\mu}\left(\frac{\partial p}{\partial x}\right) = nA_n\frac{R_n^2}{8\mu}\left(\frac{\partial p}{\partial x}\right)$$

Since,

$$A_n = \pi R_n^2$$

it follows that

$$A_1^2 = nA_n^2$$

and therefore the desired relation between A_n and n is

$$A_n = \frac{A_1}{\sqrt{n}}$$

From the continuity relation

$$A_1\overline{u}_1 = nA_n\overline{u}_n$$

and therefore the desired relation between the velocity and n is

$$\overline{u}_n = \frac{\overline{u}_1}{\sqrt{n}}$$

Using the numerical values given

$$\pi(0.015)^2 = \frac{\pi(4\times 10^{-6})^2}{\sqrt{n}}$$

and hence

$$n = 1.98\times 10^{14}$$

The actual number is much smaller than this, which implies that the velocity (and therefore, the pressure drop) is greater in the microcirculation stages.