Solution to Problem 205A

The flow coefficient, ϕ, and the head coefficient, ψ, are defined as:

$$\phi = \frac{Q}{\pi N R^3} \qquad \text{and} \qquad \psi = \frac{\Delta P}{\rho N^2 R^2}.$$

The pump designer is given required values for the flow rate, Q, and the total pressure rise, ΔP and also has desired values for ϕ_D and ψ_D:

$$\phi_D = \frac{Q}{\pi N R^3} \qquad \text{and} \qquad \psi_D = \frac{\Delta P}{\rho N^2 R^2}$$

The two unknowns in these two relations are the size of the pump R and the rotating speed N. By manipulating the equations, these parameters can be expressed in terms of the known variables. Thus:

$$N R^3 = \frac{Q}{\pi \phi_D}.$$

and

$$N^2 R^2 = \frac{\Delta P}{\rho \psi_D}$$

Eliminating N from these two equations yields

$$R^4 = \frac{\frac{Q^2}{\pi^2 \phi_D^2}}{\frac{\Delta P}{\rho \psi_D}}$$

and therefore

$$R = \left(\frac{Q}{\pi \phi_D}\right)^{\frac{1}{2}} \left(\frac{\rho \psi_D}{\Delta P}\right)^{\frac{1}{4}}$$

In addition, eliminating R yields:

$$N \left(\frac{Q}{\pi \phi_D}\right)^{\frac{3}{2}} \left(\frac{\rho \psi_D}{\Delta P}\right)^{\frac{3}{4}} = \frac{Q}{\pi \phi_D}$$

and thus

$$N = \left(\frac{\pi \phi_D}{Q}\right)^{\frac{1}{2}} \left(\frac{\Delta P}{\rho \psi_D}\right)^{\frac{3}{4}}$$