Solution to Problem 205B

(a) Labelling a point on the reservoir surface as 1 and the discharge from the tailrace as 2 the total head difference, $H_1 - H_2$, is given by

$$H_1 - H_2 = h - \frac{U^2}{2g}$$

since the pressure at both points is atmospheric and we neglect the fluid velocity at the surface of the reservoir. But it also follows that

$$H_1 - H_2 = \text{Head loss in pipes due to friction} \quad + \quad \text{Head drop through the turbine}$$

Substituting from the first equation and noting that the head loss due to friction in the pipes is equal to $kU^2/2g$ it follows that

$$\text{Head drop through the turbine} = h - \frac{U^2}{2g}(1+k)$$

(b) If the turbine were 100% efficient the power it would produce would be the total pressure drop through the turbine (total head drop times ρg) multiplied by the volume flow rate through the turbine, namely UA. Since the turbine is only 90% efficient it follows that the power, P^*, produced is

$$\text{Power produced by the turbine, } P^* = 0.9\rho g A \left[Uh - \frac{U^3}{2g}(1+k) \right]$$

(c) Differenting the above expression for the power, P^*:

$$\frac{\partial P^*}{\partial U} = 0.9\rho g A \left[h - \frac{3U^2}{2g}(1+k) \right]$$

and therefore the power will be a maximum when

$$U = \left[\frac{2gh}{3(k+1)} \right]^{\frac{1}{2}}$$