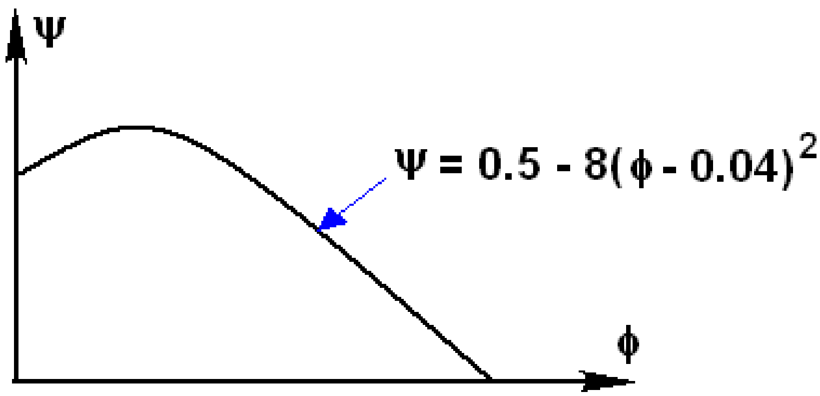
Solution to Problem 210D:

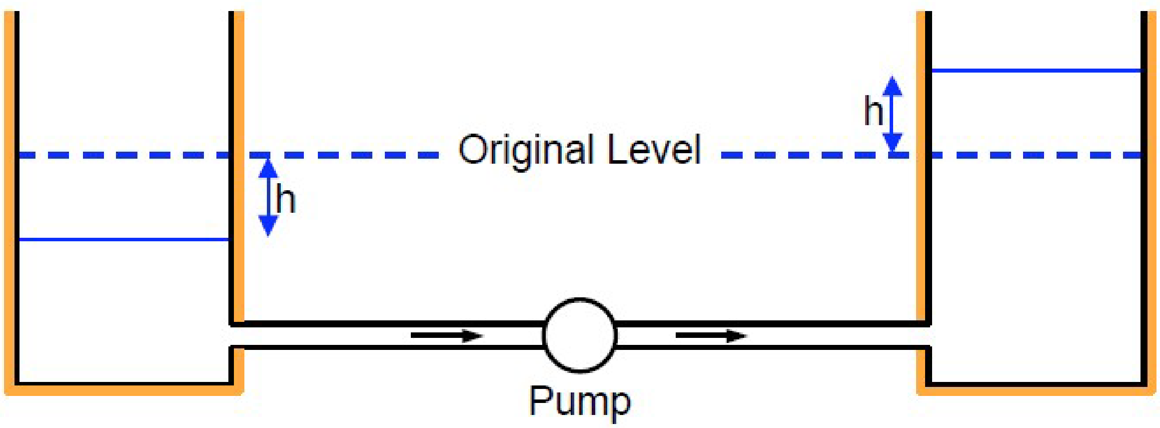
A pump has the following non-dimensional characteristic, $\psi(\phi)$: where from their definitions:



$$\phi = \frac{Q}{A\Omega R} \; ; \; \psi = \frac{g\Delta H}{\Omega^2 R^2} \tag{1}$$

where Q is the flow rate, Ω is the pump speed ($1000rpm$), R is the impeller radius ($15cm$), A is the pump discharge area ($300cm^2$), ΔH is the head rise across the pump and g is the acceleration due to gravity.

The pump is used to pump water from one tall tank or reservoir to another: beginning with the two



reservoirs levels at the same elevation. The cross-sectional area of the surface of both reservoirs is the same. The pipes connecting the reservoirs to the pump both have an internal diameter of $10cm$ and a length of $50m$; the appropriate friction factor, f, for the flow in these pipes is 0.05.

The head loss in the pipes, δH_L, is given by

$$\Delta H_L = \frac{f}{2g}\frac{L}{D}\left(\frac{Q}{A_P}\right)^2 \tag{2}$$

where A_P is the cross-sectional area of the pipes ($0.03m^2$) and f is the friction factor (0.05). Therefore teh resistance, R, of the piping into the pump and of the piping from the pump is given by

$$R = \frac{g}{Q}\Delta H_L = \frac{fL}{2D}\frac{Q}{A_P^2} = 2.026 \times 10^5 Q \tag{3}$$

The resistance of the pump, R_P, is given by $-gdH/dQ$ where

$$R_P = -g\frac{dH}{dQ} \tag{4}$$

and since

$$H = \frac{\Omega^2 R^2}{g}\left[0.5 - 8\left(\frac{Q}{AR\Omega} - 0.04\right)^2\right] \tag{5}$$

$$R_P = -\Omega R\left[-\frac{16}{A\Omega R}\left(\frac{Q}{AR\Omega} - 0.04\right)\right] = \frac{16Q}{A} - \frac{0.04R\Omega}{A} \tag{6}$$

As the head across the pump increases and the flow rate decreases the system will encounter instability when R_P becomes equal to the pipeline resistance, $2R_L$, or

$$R_P = \frac{16Q}{A} - \frac{0.04R\Omega}{A} = 2R = \frac{2fL}{2D}\frac{Q}{A_P^2} \tag{7}$$

Solving for Q this yields

$$Q_{instability} = \frac{0.04R\Omega}{A}\left[\frac{16}{A} - \frac{fL}{DA_P^2}\right]^{-1} \tag{8}$$

Substituting the applicable parameters, this yields an instability flow coefficient, ϕ, of 0.00168 and therefore an instability head coefficient, ψ, of 0.488. This, in turn, yields an instability head rise of $12.29m$. And this would occur when the difference in the reservoir levels reached $12.27m$.