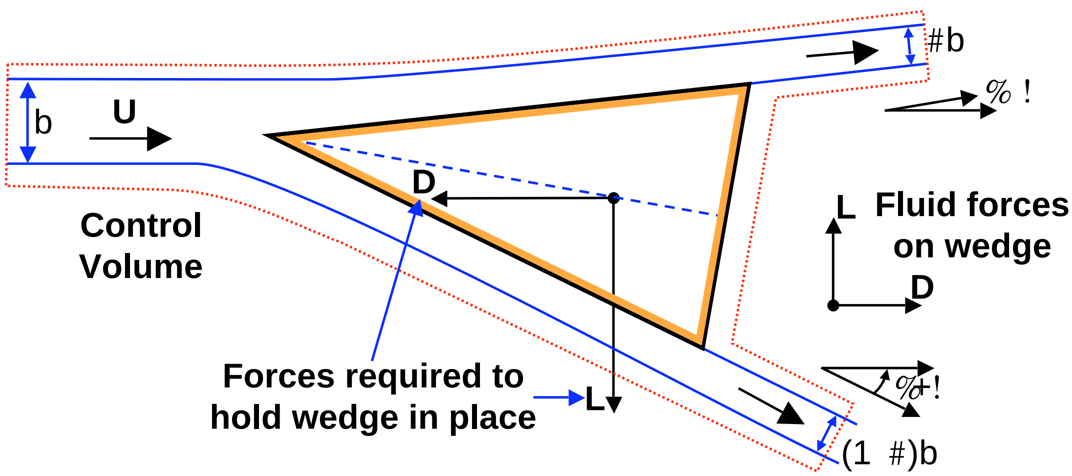
Solution to Problem 220E

We define a control volume that includes the wedge and denote the drag and lift forces on the wedge parallel and normal to the U direction by D and L as shown in the sketch:



Applying the momentum theorem in the x or U direction yields

$$
\begin{aligned}
F_x = -D &= \rho U^2 \beta b \cos(\theta - \alpha) + \rho U^2 (1-\beta) b \cos(\theta + \alpha) - \rho b U^2 \\
D &= \rho U^2 b \left[1 - \beta \cos(\theta - \alpha) - (1-\beta)\cos(\theta + \alpha)\right]
\end{aligned}
$$

Similarly, using the momentum theorem in the normal direction yields

$$L = \rho U^2 b \left[(1-\beta)\sin(\theta + \alpha) - \beta \sin(\theta - \alpha)\right]$$

Therefore the angle of attack for which L is zero is

$$\alpha = \tan^{-1}\left[(2\beta - 1)\tan(\theta)\right]$$

Also the β for zero lift is

$$\beta = \frac{1}{2}\left[1 + \tan(\alpha)\cot(\theta)\right]$$

Finally to determine whether this position is stable with respect to β, we require that, for stability, the lift must increase if the wedge is shifted downward. This requires that the lift increase as β is increased. But

$$
\begin{aligned}
\left.\frac{\partial L}{\partial \beta}\right|_{\text{at zero lift}} &= \rho b U^2 \left[-\sin(\theta + \alpha) - \sin(\theta - \alpha)\right] \\
&= \text{a } \textbf{negative} \text{ quantity for } (\theta + \alpha) < \pi \text{ and } \theta > \alpha > 0
\end{aligned}
$$

Thus we have a **unstable equilibrium**. If β is increased (body is moved down) the lift becomes negative and further pushes the body down.