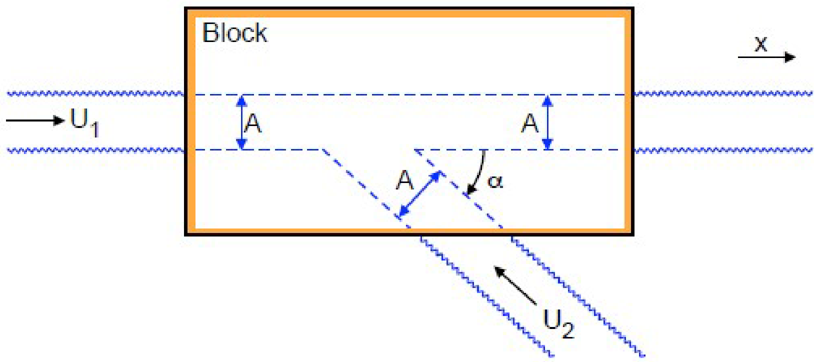
## Solution to Problem 220L:

The rectangular block shown below has holes drilled into it as indicated by the dashed lines. The holes all have the same cross-sectional area, $A$.




Consider a control volume consisting of the block. Since the fluid is incompressible the velocity of the flow out through the right hand opening is $U_1 + U_2$. Therefore the net momentum flux out of the control volume (assuming uniform velocity profiles) is

$$\rho A(U_1 + U_2)^2 \; - \; \rho A U_1^2 \; + \; (-\rho A U_2)(-U_2 \cos\alpha) \tag{1}$$

$$= \; 2\rho A U_1 U_2 \; + \; \rho A U_2^2 \; + \; \rho A U_2^2 \cos\alpha \tag{2}$$

and this must be equal to the force applied to the control volume in the $x$ direction to hold the block in place. The only forces applied which have components in the $x$ direction are the force at the $U_1$ opening due to the pressure, $p_1$, the force at the $(U_1 + U_2)$ opening due to the pressure, $p_3$, and the force applied to hold the block in place (call this $F$). Therefore

$$\text{Force in the } x \text{ direction} \; = \; (p_1 - p_3)A \; + \; F \tag{3}$$

But since the flow is frictionless Bernoulli's equation requires that

$$p_1 + \frac{1}{2}\rho U_1^2 \; = \; p_3 + \frac{1}{2}\rho (U_1 + U_2)^2 \tag{4}$$

or

$$(p_1 - p_3) \; = \; \frac{1}{2}\rho(2U_1 U_2 + U_2^2) \tag{5}$$

Therefore

$$\frac{F}{\rho A} \; = \; U_1 U_2 + \frac{U_2^2}{2} + U_2^2 \cos\alpha \tag{6}$$

and this force will be zero if

$$\frac{U_1}{U_2} \; = \; - \left[ \frac{1}{2} + \cos\alpha \right] \tag{7}$$