Solution to Problem 222A:

A turbojet engine in a wind tunnel ingests air at a velocity of $U_1 = 100m/s$ and a density of $\rho_1 = 1kg/m^3$. The velocity is uniform and the cross-sectional area of the approaching stream which enters the engine is $A_1 = 0.1m^2$. The velocity of the exhaust jet from the engine, however, is not uniform but has a velocity which varies over the cross-section according to

$$u(r) \;=\; 2U\left\{1-\frac{r^2}{r_0^2}\right\} \tag{1}$$

where the constant $U = 600m/s$ and r_0 is the radius of the jet cross-section. Radial position within the axisymmetric jet is denoted by r. The density of the exhaust jet is uniformly $\rho_2 = 0.5kg/m^3$.

(a) The average velocity is given by

$$\text{Average velocity} \;=\; \frac{1}{A}\int u\;dA \;=\; \frac{1}{\pi r_0^2}\int_0^{r_0} 2U\left\{1-\frac{r^2}{r_0^2}\right\}\;dr \;=\; U \;=\; 600m/s \tag{2}$$

(b) Continuity requires that $\rho_1 U_1 A_1 = \rho_2 U A_2$ and therefore $A_2 = \rho_1 U_1 A_1/\rho_2 U$ so $A_2 = 1/30\ m^2$. The momentum theorem requires that the thrust produced by the jet engine, F, is equal to the net momentum flux out of the engine or

$$F \;=\; -\rho_1 U_1^2 A_1 + \int_0^{r_0} 2\pi\rho_2 u^2(r) r\;dr \;=\; -\rho_1 U_1^2 A_1 + \frac{4}{3}\rho_2 A_2 U^2 \tag{3}$$

and therefore $F = 7000N$.

(c) If the discharge velocity were uniform and equal to U then

$$F \;=\; -\rho_1 U_1^2 A_1 + \rho_2 A_2 U^2 \;=\; 5000N \tag{4}$$