Solution to Problem 240B

If the boundary layer is like that of a flat plate (for which $dU/dx = 0$) then the Blasius solution applies and

$$\begin{aligned}\delta_D &= 1.72 \left(\frac{\nu x}{U}\right)^{\frac{1}{2}} \\ &= 1.72 \left(\frac{2.5\times 10^{-6}\ x}{1.0}\right)^{\frac{1}{2}}\ m \\ &= 2.7\times 10^{-3}\ x^{\frac{1}{2}}\ m\end{aligned}$$

The effect of this displacement thickness is to yield a volume flow rate in the tube which is the same as the volume flow rate of a *uniform* stream in a tube of radius $(R-\delta_D)$ where R is the actual radius of the tube. Therefore the velocity of the flow outside the boundary layer is not 1 m/s but U_x where

$$U\pi R^2 = U_x \pi \left(R-\delta_D\right)^2$$

At $x = 200\ m$, where $\delta_D = 0.038\ m$ this yields

$$U_x = \frac{U}{\left(1-\delta_D/R\right)^2} = 1.39\ m/s$$

Since Bernoulli's equation applies outside the boundary layer the pressure at $x = 200\ m$ is related to the pressure at the inlet $(x = 0\ m)$ by

$$\begin{aligned}p_{x=200\ \mathrm{m}} - p_{x=0\ \mathrm{m}} &= \frac{1}{2}\rho\left[U^2 - U_x^2\right] \\ &= -0.57\ \mathrm{Pa}\end{aligned}$$

where $\rho = 1.2\ kg/m^3$.

In order to proceed to a more accurate solution that takes this pressure and velocity gradient into account, one might approximately estimate the value of the Falkner-Skan m from

$$m = \frac{x}{U}\frac{dU}{dx} \approx \frac{U_{x=200\ m}}{U} - 1 = 0.39$$

and then use the Falkner-Skan solution for this value of m instead of the Blasius solution to evaluate the displacement thickness.