Solution to Problem 267A

The Rayleigh equation is

$$(\omega - ku)\left[f'' - k^2 f\right] + ku'' f = 0$$

where $u(y)$ is the velocity profile of the unidirectional flow, the prime denotes differentiation with respect to y and the form of the disturbance in the flow is assumed to be a perturbation in the streamfunction, ψ, of the form

$$\psi = f(y) \exp i(kx - \omega t)$$

where the as-yet-undetermined amplitude of the disturbance, $f(y)$, is a function only of y, k and ω are the wavenumber and radian frequency of the disturbance. For the spatial stability problem ω is real and $k = k_R + ik_I$ so that the disturbance radian frequency is ω, the disturbance wavelength is $2\pi/k_R$ and the disturbance amplification rate is $-k_I$.

A convenient non-dimensional version of the Rayleigh equation is

$$\left[\frac{\omega\delta}{U} - k\delta\frac{u}{U}\right]\left[\frac{d^2(f/\delta U)}{d(y/\delta)^2} - (k\delta)^2(f/\delta U)\right] + k\delta\frac{d^2(u/U)}{d(y/\delta)^2}\frac{f}{\delta U} = 0$$

where δ is the boundary layer thickness and U is the velocity just outside the boundary layer. With this choice of non-dimensionalization, the non-dimensional frequency is $\omega\delta/U$, the non-dimensional wavelength is $2\pi/k_R\delta$ and the non-dimensional amplification rate is $-k\delta$. As a result the non-dimensional amplification rate will be a function of the non-dimensional frequency and the non-dimensional wavelength.

If the viscous terms are included the resulting Orr-Sommerfeld equation is identical to the above Rayleigh equation except that it includes the following additional viscous term on the left-hand side:

$$-i\frac{\nu}{\delta U}\left[\frac{d^4(f/\delta U)}{d(y/\delta)^4} - 2(k\delta)^2\frac{d^2(f/\delta U)}{d(y/\delta)^2} + (k\delta)^4(f/\delta U)\right]$$

and therefore the additional Reynolds number parameter, $Re = \nu/\delta U$, is introduced.