Solution to Problem 270C

There are two analytical tools available to find the average velocity in this pipe flow. First, the friction factor gives

$$f = \frac{D\left(-\frac{dp}{dx}\right)}{\frac{1}{2}\rho V^2}$$

$$V_f = \sqrt{\frac{D\left(-\frac{dp}{dx}\right)}{\frac{1}{2}\rho f}} = \sqrt{\frac{0.5\ m\left(\frac{1\ kg/m\ s^2}{50\ m}\right)}{\frac{1}{2}(1.2\ kg/m^3)f}} = \sqrt{\frac{1}{60f}}\ m/s$$

Second, the definition of the Reynolds number yields

$$Re = \frac{DV}{\nu}$$

$$V_{Re} = \frac{Re\ \nu}{D} = \frac{2.3\times 10^{-6}\ m^2/s}{0.5\ m}\mathrm{Re} = 4.6\times 10^{-6}\ \mathrm{Re}\ m/s$$

Thus there are two equations and three unknowns (f, Re, $V = V_f = V_{Re}$). To solve the problem, one must guess either the Reynolds number or the friction factor and then use the Moody chart to iterate toward the correct answer. If we start with a guessed value of the Reynolds number of 6×10^4, then the Moody chart yields $f = 0.02$ and the values of V_f and V_{Re} on the first line follow from the equations above. It also floows therefore that the Reynolds number must actually be greater than 6×10^4 and hence the second iteration on the second line. The other iterations then follow until we find a Reynolds number which yields equal values of V_f and V_{Re} as follows:

Iteration	Re	f	V_f (m/s)	V_{Re} (m/s)
1	6×10^4	0.02	0.912	0.276
2	2×10^5	0.0155	1.04	0.92
3	3×10^5	0.014	1.09	1.38
4	2.5×10^5	0.015	1.05	1.15
5	2.4×10^5	0.015	1.05	1.104
6	2.3×10^5	0.0151	1.047	1.058

Therefore,

$$V \simeq 1.05\ \mathrm{m/s}$$