Solution to Problem 272C:

In this steady flow since the velocities do not change with x it follows that the forces acting on a fluid element of length dx must balance and therefore the net pressure force in the x direction, namely $h(-dp/dx)dx$, must be equal to the frictional forces at the walls, namely $2\tau_w dx$ so that

$$\tau_w \;=\; \frac{h}{2}\left(-\frac{dp}{dx}\right) \tag{1}$$

and therefore using the definitions

$$f \;=\; \frac{2h}{\rho V^2}\left(-\frac{dp}{dx}\right) \;\; ; \;\; u_\tau \;=\; \sqrt{\tau_w/\rho} \tag{2}$$

it follows that

$$\frac{u_\tau}{V} \;=\; \left(\frac{f}{4}\right)^{1/2} \tag{3}$$

The volumetric average velocity, V, is

$$V \;=\; \frac{2}{h}\int_0^{h/2} \overline{u}(y)\; dy \tag{4}$$

where $\overline{u}(y)$ is the average fluid velocity at a distance y from the midline of the flow (a line of symmetry). Then substituting the universal turbulent velocity profile

$$\overline{u}(y) \;=\; 5.75\log_{10}\left(\frac{yu_\tau}{\nu}\right) + 5.5 \tag{5}$$

into the above integral and integrating we obtain the following relation connecting the friction factor, f, and the Reynolds number of the flow, $Re = hV/\nu$:

$$(f)^{-1/2} \;=\; 2.88\log_{10}\left(f^{1/2}Re\right) - 0.462 \tag{6}$$