Solution to Problem 276B

This is a Couette flow in which the total shear stress, σ, is given as usual by

$$\sigma = \mu \frac{d\bar{u}}{dy} - \rho \overline{u'v'}$$

where the first term on the right hand side is the laminar component and the second term is the Reynolds' stress. If the magnitudes of the unsteady velocities, u' and v', are both estimated to be $U\epsilon y/h^2$ then

$$\sigma = \mu \frac{d\bar{u}}{dy} + \rho \left(\frac{U\epsilon y}{h^2} \right)^2$$

and since σ does not vary with y in any Couette flow it follows that

$$\frac{d\bar{u}}{dy} = \frac{\sigma}{\mu} - \frac{\rho U^2 \epsilon^2}{\mu h^4} y^2$$

and we can integrate this to obtain

$$\bar{u} = \frac{\sigma y}{\mu} - \frac{\rho U^2 \epsilon^2}{\mu h^4} \frac{y^3}{3}$$

where we have used $\bar{u} = 0$ on $y = 0$ to eliminate the integration constant. therefore

$$U = \frac{\sigma h}{\mu} - \frac{\rho U^2 \epsilon^2}{3 \mu h}$$

and

$$\sigma = \frac{\mu U}{h} + \frac{\rho U^2 \epsilon^2}{3 h^2}$$

The "effective" viscosity of the film, $\mu*$, will be given by $\mu* = \sigma h/U$ and therefore

$$\frac{\mu*}{\mu} = 1 + \frac{U \epsilon^2}{3 \nu h}$$