Solution to Problem 276C

The Reynolds number based on the length, L, and the speed, U, of the yacht is

$$\mathrm{Re}_L = \frac{UL}{\nu} = \frac{(4)(16.8)}{10^{-6}} = 6.72 \times 10^7$$

The coefficient of total drag on the yacht (idealized as a plate) is

$$C_D = \frac{\mathrm{Drag}}{\frac{1}{2}\rho U^2 L w} = \frac{0.074}{\mathrm{Re}_L^{\frac{1}{5}}}$$

where L and w are the length and total width of the plate ($w = 2\ m$). Re-arranging to solve for the drag

$$\mathrm{Drag} = \left(\frac{0.074}{\mathrm{Re}^{\frac{1}{5}}}\right)\frac{1}{2}\rho U^2 L w = \left[\frac{0.074}{(6.72\times 10^7)^{\frac{1}{5}}}\right]\frac{1}{2}\left(10^3\right)(4)^2(16.8)(2) = 540\ \mathrm{N}$$

The surface would be "hydraulically smooth" if the roughness, ϵ, is less than the laminar sublayer thickness, δ_{LSL}, where

$$\frac{\delta_{LSL} u_\tau}{\nu} = 5, \qquad u_\tau = \sqrt{\frac{\tau_w}{\rho}}$$

The coefficient of local skin friction for a turbulent boundary layer on a flat plate is

$$\frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.058}{\left(\frac{Ux}{\nu}\right)^{\frac{1}{5}}}$$

Solving this expression for τ_w at $x = 8.4$ m (the mid-length of the yacht) and inserting it into the definition for u_τ yields

$$(u_\tau)_{\text{mid-length}} = \sqrt{\frac{(\tau_w)_{\text{mid-length}}}{\rho}} = \left[\frac{1}{2}U^2 \frac{0.058}{\left(\frac{U(8.4)}{\nu}\right)^{\frac{1}{5}}}\right]^{\frac{1}{2}} = 0.119\ \mathrm{m/s}$$

Consequently for the surface of the yacht to be hydraulically smooth requires that

$$\epsilon \leq \delta_{\mathrm{LSL}} = \frac{5\nu}{u_\tau} = \frac{5(10^{-6})}{0.119} = 42 \times 10^{-6}\ \mathrm{m}$$

and so the maximum admissible roughness is $42 \times 10^{-6} m$.