Solution to Problem 292C

Find the minimum glide angle for the airplane.

At equilibrium, the lift and drag forces must be balanced by the weight of the airplane.

$$\Rightarrow \tan\beta = \frac{D_{tot}}{L_{tot}} = \frac{(C_D)_{tot}}{(C_L)_{tot}}$$

The total lift coefficient, $(C_L)_{tot}$, is the 2-D lift coefficient with the correction for finite aspect ratio wings.

$$\begin{aligned}(C_L)_{tot} &= (C_L)_{2D} + \Delta C_L \\ &= (C_L)_{2D}\left[1 - \frac{1}{1+\frac{A_R}{2}}\right] \\ &= (C_L)_{2D}\left[\frac{A_R}{2+A_R}\right]\end{aligned}$$

Since drag on the rest of the airplane is given as four times the drag on the wings, the total drag coefficient, $(C_D)_{tot}$, is five times the corrected wing drag coefficient.

$$(C_D)_{tot} = 5(C_D)_{wing} = 5\left[(C_D)_{2D} + \frac{(C_L)_{2D}^2}{\pi A_R}\right]$$

Substituting these relations for the lift and drag coeffients into the expression for the glide angle, β, we get an equation for the glide angle in terms of the 2-D lift and drag coefficients (which can be read from the given plot) and the aspect ratio.

$$\tan\beta = \frac{5(2+A_R)}{A_R}\left[\frac{(C_D)_{2D}}{(C_L)_{2D}} + \frac{(C_L)_{2D}}{\pi A_R}\right]$$

Note: If there was no ΔC_D then we could find the minimum glide angle by simply minimizing $(C_D/C_L)_{2D}$. This could be done by finding the slope of the line through the origin which just touches the curve of C_L versus C_D. On account of the drag correction, we must solve by trial and error.

$(C_L)_{2D}$	$(C_D)_{2D}$	$\tan\beta$	β
1.0	0.008	0.239	13.44°
0.6	0.0065	0.180	10.18°
0.4	0.0064	0.172	9.78°
0.2	0.0064	0.230	12.96°

So the minimum glide angle is approximately 9.8°.