Solution to Problem 295A

Consider a simplified view of the propulsion of a paddle steamer:

1. Find a relation between the propulsion velocity, U and the relative paddle velocity, V. Since the steamer isn't accelerating, the forces must match. The drag force, $D = C_D(\frac{1}{2}\rho U^2)A$. Thus,

$$C_{DP}\frac{1}{2}\rho(V-U)^2 A_P = C_{DH}\frac{1}{2}\rho U^2 A_H$$
$$V = \left(1+\sqrt{\frac{C_{DH}A_H}{C_{DP}A_P}}\right)U$$

or

$$U = \frac{V}{1+\sqrt{\frac{C_{DH}A_H}{C_{DP}A_P}}}$$

2. The efficiency is the ratio of useful work done to total work done.

$$\eta = \frac{C_{DH}\frac{1}{2}\rho U^3 A_H}{C_{DH}\frac{1}{2}\rho U^3 A_H + C_{DP}\frac{1}{2}\rho (V-U)^3 A_P} = \frac{1}{\sqrt{\frac{C_{DH}A_H}{C_{DP}A_P}}+1}$$

If C_{DH} and C_{DP} are both of order unity, to obtain reasonable efficiencies $\frac{A_H}{A_P}$ should be made as small as possible. As it's difficult to make A_H small with traditional ships, A_P should be made as large as possible. If one looks at the development of paddle steamers, the size of the paddles was indeed made larger and larger through time. As the paddle cannot be made infinitely large, the efficiency for these vessels was never very big and they were eventually overtaken by propeller boats.