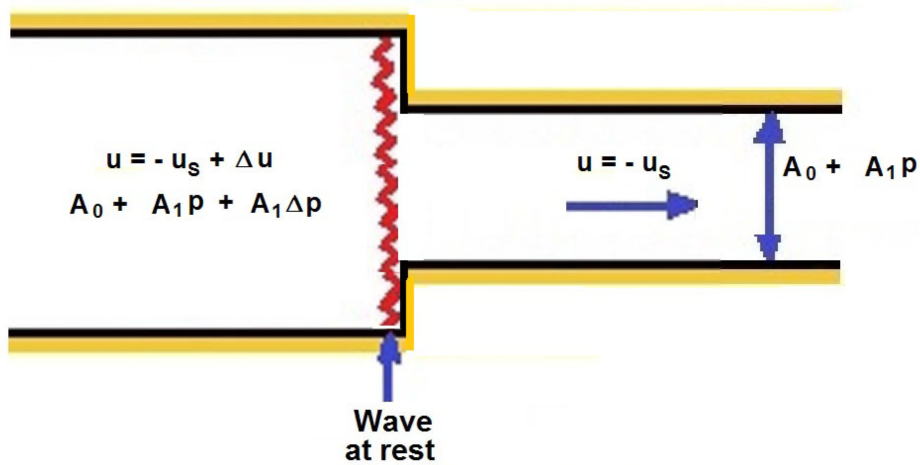
Solution to Problem 310A:

In a frame of reference traveling with the wave at velocity, u_S:



The conditions ahead of the wave in the undisturbed pipe the cross-sectional area: $A = A_0 + A_1 p$, the fluid velocity relative to the wave, $u = -u_S$, the fluid density is ρ and the pressure is p while the conditions behind the wave are: $A = A_0 + A_1 p + A_1 \Delta p$, the relative fluid velocity $u = -u_S + \Delta u$, the fluid density is ρ (since the fluid is incompressible) and the pressure is $p = p + \Delta p$.

Therefore continuity requires that

$$\rho(A_0 + A_1 p + A_1 \Delta p)(-u_S + \Delta u) \;=\; \rho(A_0 + A_1 p)(-u_S) \tag{1}$$

and so

$$\Delta p \;=\; \frac{(A_0 + A_1 p)\Delta u}{A_1 u_S} \tag{2}$$

The momentum theorem requires that

$$\Delta p(A_0 + A_1 p) \;=\; \rho(A_0 + A_1 p)u_S^2 - \rho(A_0 + A_1 p + A_1 \Delta p)(-u_S + \Delta u)^2 \tag{3}$$

and using the continuity relation

$$\Delta p(A_0 + A_1 p) \;=\; \rho(A_0 + A_1 p)u_S \Delta u \tag{4}$$

Eliminating Δu from the second and fourth equations:

$$u_S^2 \;=\; \frac{(A_0 + A_1 p)}{\rho A_1} \tag{5}$$

or since $A_1 p \ll A_0$:

$$u_S \;=\; \left(\frac{A_0}{\rho A_1}\right)^{1/2} \tag{6}$$