Solution to Problem 321A

Show that the flow through the hole of cross-sectional area, A^*, is choked.

$$\frac{p_0}{p_E} = \frac{10^7}{10^5} = 100$$

If we assume isentropic flow through the hole, the Mach number at the exit of the hole can be found: $M_E = 3.7$. Since the flow transitions from subsonic ($M_0 = 0$) in the reservoir to supersonic ($M_E = 3.7$) at the exit, the flow must pass through a throat. For $M = 1$ at the throat, $\frac{p_0}{p^*} = 1.893$. Choking will occur for $\frac{p_0}{p_A} > 1.893$ so this flow is choked.

Find the mass flow rate out of the reservoir and the pressure and velocity of the flow in the throat formed by the hole.

From above, $M = 1$ at the throat gives $\frac{p_0}{p^*} = 1.893$

$$p^* = \frac{10^7}{1.893} = 5.28MPa$$

The velocity at the throat can be calculated from the Mach number by finding the sound speed at the throat. The temperature at the throat can be found from the reservoir temperature using the equations for isentropic flow: $\frac{T_0}{T^*} = 1.2$.

$$\begin{aligned} u^* &= M^* \sqrt{\gamma R T^*} \\ &= 1\sqrt{\gamma R \frac{T_0}{1.2}} = 314.6\frac{m}{s} \end{aligned}$$

We calculate the mass flow rate out of the reservoir as the mass flow rate through the throat. This requires the density at the throat, which can be found from the pressure and temperature at the throat with the equation of state, $p^* = \rho^* R T^*$.

$$\begin{aligned} \dot{m} &= \rho^* u^* A* \\ &= \frac{p^*}{RT^*} u^* A^* = 4.7\frac{kg}{s} \end{aligned}$$