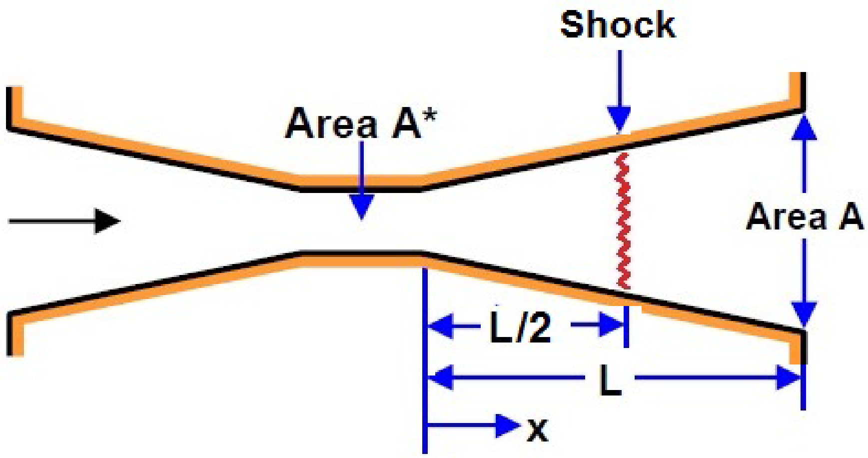
Solution to Problem 332B:

A crude de Laval nozzle with a throat area, A^*, and a diffuser exit area 16 times larger ($A = 16A^*$) is made using a straight-sided conical diffuser as indicated below: The nozzle is supplied from an air reservoir



($\gamma = 1.4$) of pressure, p_0; the external pressure of the air downstream of the diffuser exit is p_E. We seek the ratio p_E/p_0 at which a normal shock will form half-way along the diffuser, that is to say at $x/L = 0.5$.

First a little geometry. Since the shock is halfway along the diffuser, it follows that the area of the flow at the shock, $A_S = 6.25A^*$. Then using the shock wave table for $A_S/A^* = 6.25$ we find that the Mach number, M_1, of the flow just upstream of the shock is $M_1 = 3.39$ and the pressure just upstream of the shock, p_1, is related to the total pressure of the flow at that point, p_{01}, by $p_1/p_{01} = 0.01596$. Since the total pressure in the isentropic flow upstream of the shock is the same everywhere upstream of the shock it follows that p_{01} is also the pressure in the reservoir, p_0.

From the shock wave table if $M_1 = 3.39$, then the Mach number of the flow just downstream of the shock is $M_2 = 0.456$ and the pressure just downstream of the shock, p_2, is given by $p_2/p_1 = 13.32$. Therefore, from the isentropic flow table $A_S/A_2^* = 1.44$ where A_2^* is the "throat area" of the flow downstream of the shock. In addition, from the isentropic flow table $p_2/p_{02} = 0.867$ where p_{02} is the total pressure downstream of the shock which is the same everywhere downstream of the shock.

It follows that since $A_2^* = A_S/1.44$ and $A = 16A^*$ then $A = 3.67A_2^*$ and using this area ratio in the isentropic flow, it transpires that the exit pressure, p_E, is given by $p_E/p_{02} = 0.981$.

Finally then

$$\frac{p_E}{p_0} = \frac{p_E}{p_{01}} = \frac{p_1}{p_{01}}\frac{p_2}{p_1}\frac{p_{02}}{p_2}\frac{p_E}{p_{02}} = \frac{0.0159 \times 13.32 \times 0.981}{0.867} = 0.241 \qquad (1)$$