Solution to Problem 334A

The normal shock relations give the drop in pressure across the shock. The upstream pressure and Mach number are p_A and M, respectively. We denote the downstream pressure and Mach number as p_2 and M_2.

$$\frac{p_2}{p_A} = 1 + \frac{2\gamma}{\gamma+1}\left(M^2 - 1\right)$$

The pressure varies from the back of the shock to the stagnation point by the isentropic flow relations. Note that $p_s = p_0$.

$$\frac{p_0}{p_2} = \frac{p_s}{p_2} = \left(1 + \frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma}{\gamma-1}}$$

M_2 can be related to the upstream Mach number through the normal shock relations.

$$M_2^2 = \frac{1 + \frac{\gamma-1}{2}M^2}{\gamma M^2 - \frac{\gamma-1}{2}}$$

Combining these three relations gives the pressure ratio, $\frac{p_s}{p_A}$ is terms of the speed of the aircraft, M.

$$\begin{aligned}\frac{p_s}{p_A} = \frac{p_s}{p_2}\frac{p_2}{p_A} &= \left[1 + \frac{2\gamma}{\gamma+1}\left(M^2-1\right)\right]\left(1 + \frac{\gamma-1}{2}M_2^2\right)^{\frac{\gamma}{\gamma-1}} \\ &= \left[1 + \frac{2\gamma}{\gamma+1}\left(M^2-1\right)\right]\left[1 + \frac{\gamma-1}{2}\left(\frac{1+\frac{\gamma-1}{2}M^2}{\gamma M^2 - \frac{\gamma-1}{2}}\right)\right]^{\frac{\gamma}{\gamma-1}}\end{aligned}$$

$$\Rightarrow \frac{p_s}{p_A} = \left(\frac{2\gamma}{\gamma+1}M^2 - \frac{\gamma-1}{\gamma+1}\right)\left[1 + \frac{\gamma-1}{2}\left(\frac{2+(\gamma-1)M^2}{2\gamma M^2 - (\gamma-1)}\right)\right]^{\frac{\gamma}{\gamma-1}}$$