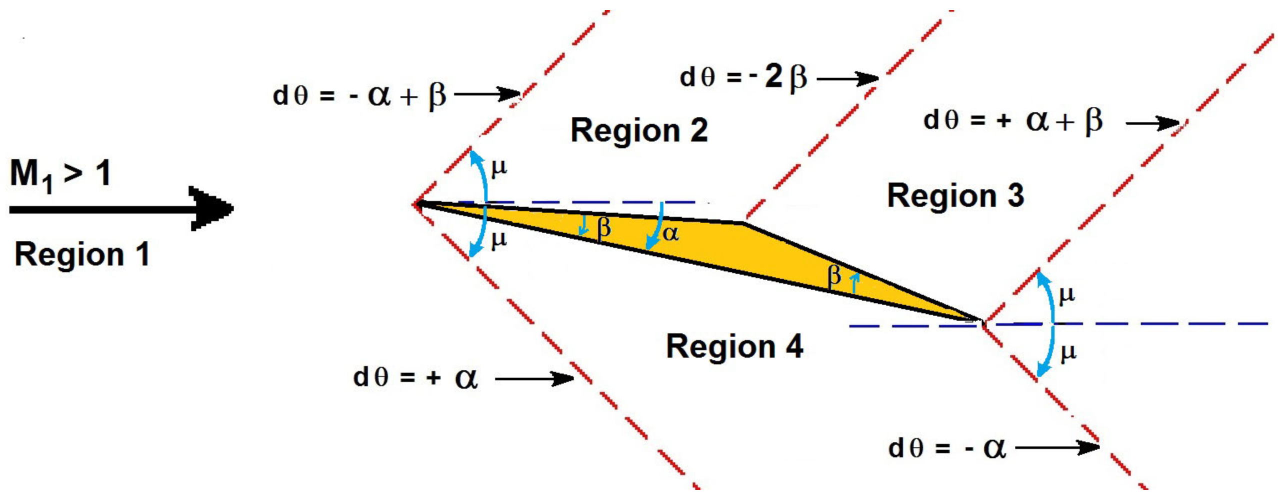
Solution to Problem 340C:

Using supersonic flow theory for small angles of turn to find the lift and drag coefficients for supersonic flow past a thin airfoil of the following shape:



The angle of expansion between region 1 and region 2 is $(\alpha - \beta)$ so

$$\frac{p_2 - p_1}{p_1} = \frac{\gamma M^2}{(M^2-1)^{1/2}} \; (-(\alpha-\beta)) \tag{1}$$

The angle of expansion between region 1 and region 3 is $(\alpha + \beta)$ so

$$\frac{p_3 - p_1}{p_1} = \frac{\gamma M^2}{(M^2-1)^{1/2}} \; (-(\alpha+\beta)) \tag{2}$$

The angle of compression between region 1 and region 4 is α so

$$\frac{p_4 - p_1}{p_1} = \frac{\gamma M^2}{(M^2-1)^{1/2}} \; (\alpha) \tag{3}$$

The lift, L, and drag, D, are given by

$$L = -\frac{A}{2} p_1 \cos(\alpha - \beta) - \frac{A}{2} p_3 \cos(\alpha+\beta) + p_4 \cos\alpha \tag{4}$$

$$D = -\frac{A}{2} p_1 \sin(\alpha - \beta) - \frac{A}{2} p_3 \sin(\alpha+\beta) + p_4 \sin\alpha \tag{5}$$

and substituting for p_2, p_3 and p_4 and assuming small α and β:

$$L = \frac{A}{2} \frac{\gamma M^2 p_1}{(M^2-1)^{1/2}} (4\alpha) \tag{6}$$

$$D = \frac{A}{2} \frac{\gamma M^2 p_1}{(M^2-1)^{1/2}} (4\alpha^2 + 2\beta^2) \tag{7}$$

and since $\gamma M^2 p_1 = \rho_1 u_1^2$ and $C_L = 2L/A\rho_1 u_1^2$ and $C_D = 2D/A\rho_1 u_1^2$ then

$$C_L = \frac{4\alpha}{(M^2-1)^{1/2}} \quad \text{and} \quad C_D = \frac{(4\alpha^2+2\beta^2)}{(M^2-1)^{1/2}} \tag{8}$$

noting that β is a measure of the thickness of the foil, increasing the thickness increases the drag but leaves the lift unchanged.

The lift/drag ratio is given by $4\alpha/(4\alpha^2+2\beta^2)$ and for a given , fixed β is a maximum when $\alpha = \beta/2^{1/2}$.