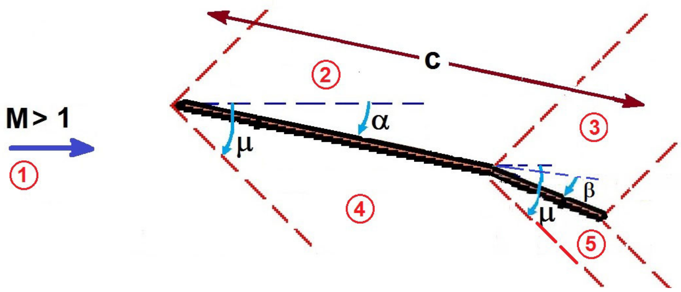
Solution to Problem 340G:

A flat plate foil is fitted with a flap hinged at the 3/4 chord point as follows:



The oncoming stream is supersonic, $M > 1$, and the forward section of the foil is set at an angle of attack, α. The flap is inclined at an angle β relative to the forward section. The flow is to be analyzed using the supersonic theory for small angles of turn.

Let $K = \gamma M^2/(M^2-1)^{1/2}$ for convenience. Then the pressures in the regions 1, 2, 3, 4 and 5 above are

$$\frac{p_2-p_1}{p_1} = -K\alpha \quad \text{and} \quad \frac{p_4-p_1}{p_1} = +K\alpha \tag{1}$$

$$\frac{p_3-p_1}{p_1} = -K(\alpha+\beta) \quad \text{and} \quad \frac{p_5-p_1}{p_1} = +K(\alpha+\beta) \tag{2}$$

Therefore the pressure difference across the forward section and the flap are

$$\frac{p_4-p_2}{p_1} = 2K\alpha \quad \text{and} \quad \frac{p_5-p_3}{p_1} = 2K(\alpha+\beta) \tag{3}$$

and the forces (per unit depth normal to the sketch) on the forward section and the flap are

$$2K\alpha p_1 \times \frac{3C}{4} \quad \text{and} \quad 2K(\alpha+\beta)p_1 \times \frac{C}{4} \tag{4}$$

and the lift per unit depth, L, is

$$L = \frac{Kp_1c}{2}(3\alpha+\alpha+\beta) \tag{5}$$

and the lift coefficient, C_l, is

$$C_L = \frac{(4\alpha+\beta)}{(M^2-1)^{1/2}} \tag{6}$$

It follows that the flap lift slope, $dC_L/d\beta$, is

$$\frac{dC_L}{d\beta} = \frac{1}{(M^2-1)^{1/2}} \tag{7}$$

From a practical point of view it is useful to have a flap sensitivity which is independent of the angle of attack, α.