Solution to Problem 344A:

Consider first the appropriate **incompressible** velocity potential that must involve terms with $\sin kx$ or $\cos kx$:

$$\phi \;=\; Ae^{-ky}\sin kx + Be^{-ky}\cos kx \tag{1}$$

where terms involving e^{+ky} are clearly unacceptable and have been omitted.

Then in the compressible flow the equivalent velocity potential will follow from the **Prandtl-Glauert** transformation in which y is replaced by $(1-M^2)^{1/2}y$:

$$\phi \;=\; Ae^{-ky(1-M^2)^{1/2}}\sin kx + Be^{-ky(1-M^2)^{1/2}}\cos kx \tag{2}$$

Alternatively this could be obtained by separation of variables applied to the equation that ϕ must satisfy, namely,

$$(1-M^2)\frac{\partial^2\phi}{\partial x^2} \;+\; \frac{\partial^2\phi}{\partial y^2} \;=\; 0 \tag{3}$$

Adding the uniform stream potential $\phi = Ux$ to the above yields the appropriate form for the velocity potential of this flow:

$$\phi \;=\; Ux + Ae^{-ky(1-M^2)^{1/2}}\sin kx + Be^{-ky(1-M^2)^{1/2}}\cos kx \tag{4}$$

The boundary condition that must be satisfied at the wavy surface is

$$v \;=\; \left(\frac{\partial\phi}{\partial y}\right)_{y=0} \;=\; U\frac{dh}{dx} \tag{5}$$

and so it follows that $A = 0$ and $B = -Ua/(1-M^2)^{1/2}$. Therefore the final result is

$$\phi \;=\; Ux - \frac{Ua}{(1-M^2)^{1/2}}e^{-ky(1-M^2)^{1/2}}\cos kx \tag{6}$$