Solution to Problem 354B

1.) Find the inclination angle, ϕ, of the reflected shock.

Based on the given oblique shock wave angle and the oncoming Mach number, M_1, we can find the angle through which the flow is deflected by the airplane from the θ-β-M relation:

$$\tan\theta = 2\cot\beta\left[\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma+\cos 2\beta)+2}\right]$$

$$\Rightarrow \theta = 17.68^\circ$$

Finding the incoming Mach number normal to the surface of the oblique shock:

$$M_{1n} = M_1\sin\beta = 1.607$$

We use the normal shock relations to find the Mach number behind the shock.

$$M_{2n}^2 = \frac{1+\frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}} = 0.444$$

$$\Rightarrow M_{2n} = 0.66635$$

$$M_2 = \frac{M_{2n}}{\sin(\beta-\theta)} = 1.75$$

The reflected shock will turn the flow so that it is again parallel with the ground. We use the deflection angle, $\theta = 17.68$, and $M_2 = 1.75$ on the graph of oblique shock properties to find the reflected oblique shock wave angle, β_2.

$$\Rightarrow \beta_2 = 60.46^\circ$$

From the geometry:

$$\phi + \theta = \beta_2$$

$$\Rightarrow \phi = \beta_2 - \theta = 60.46^\circ - 17.68^\circ = 42.8^\circ$$

2.) Find the Mach number downstream of the reflected shock.

The oncoming Mach number normal to the reflected shock is:

$$M_{2n} = M_2\sin\beta_2 = 1.53$$

Using the normal shock relations to find the normal Mach number downstream of the reflected shock:

$$M_{3n}^2 = \frac{1+\frac{\gamma-1}{2}M_{2n}^2}{\gamma M_{2n}^2 - \frac{\gamma-1}{2}} = 0.4789$$

$$\Rightarrow M_{3n} = 0.69$$

$$M_3 = \frac{M_{3n}}{\sin(\beta_2-\theta)} = 1.02$$