Stability of Vapor/Gas Bubbles

Apart from the characteristic bubble growth and collapse processes discussed in the last section, it is also important to recognize that the equilibrium condition

$$p_V - p_\infty + p_{Ge} - \frac{2S}{R_e} = 0 \tag{Nge1}$$

may not always represent a *stable* equilibrium state at $R = R_e$ with a partial pressure of gas p_{Ge}.

Consider a small perturbation in the size of the bubble from $R = R_e$ to $R = R_e(1+\epsilon)$, $\epsilon \ll 1$ and the response resulting from the Rayleigh-Plesset equation. Care must be taken to distinguish two possible cases:

(i) The partial pressure of the gas remains the same at p_{Ge}.

(ii) The mass of gas in the bubble and its temperature, T_B, remain the same.

From a practical point of view the Case (i) perturbation is generated over a length of time sufficient to allow adequate mass diffusion in the liquid so that the partial pressure of gas is maintained at the value appropriate to the concentration of gas dissolved in the liquid. On the other hand, Case (ii) is considered to take place too rapidly for significant gas diffusion. It follows that in Case (i) the gas term in the Rayleigh-Plesset equation (Ngd2) is p_{Ge}/ρ_L whereas in Case (ii) it is $p_{Ge}R_e^{3k}/\rho_L R^{3k}$. If n is defined as zero for Case (i) and $n = 1$ for Case (ii) then substitution of $R = R_e(1+\epsilon)$ into the Rayleigh-Plesset equation yields

$$R\frac{d^2R}{dt^2} + \frac{3}{2}\left(\frac{dR}{dt}\right)^2 + \frac{4\nu_L}{R}\frac{dR}{dt} = \frac{\epsilon}{\rho_L}\left\{\frac{2S}{R_e} - 3nkp_{Ge}\right\} \tag{Nge2}$$

Note that the right-hand side has the same sign as ϵ if

$$\frac{2S}{R_e} > 3nkp_{Ge} \tag{Nge3}$$

and a different sign if the reverse holds. Therefore, if the above inequality holds, the left-hand side of equation (Nge2) implies that the velocity and/or acceleration of the bubble radius has the same sign as the perturbation, and hence the equilibrium is *unstable* since the resulting motion will cause the bubble to deviate further from $R = R_e$. On the other hand, the equilibrium is stable if $np_{Ge} > 2S/3R_e$.

First consider Case (i) which must always be *unstable* since the inequality (Nge3) always holds if $n = 0$. This is simply a restatement of the fact (discussed in section (Ngi)) that, if one allows time for mass diffusion, then all bubbles will either grow or shrink indefinitely.

Case (ii) is more interesting since, in many of the practical engineering situations, pressure levels change over a period of time that is short compared with the time required for significant gas diffusion. In this case a bubble in stable equilibrium requires

$$p_{Ge} = \frac{m_G T_B \mathcal{R}_G}{\frac{4}{3}\pi R_e^3} > \frac{2S}{3kR_e} \tag{Nge4}$$

where m_G is the mass of gas in the bubble and $\mathcal{R}_G$ is the gas constant. Indeed for a *given* mass of gas there exists a critical bubble size, R_c, where

$$R_c = \left\{\frac{9km_GT_B\mathcal{R}_G}{8\pi S}\right\}^{1/2} \tag{Nge5}$$

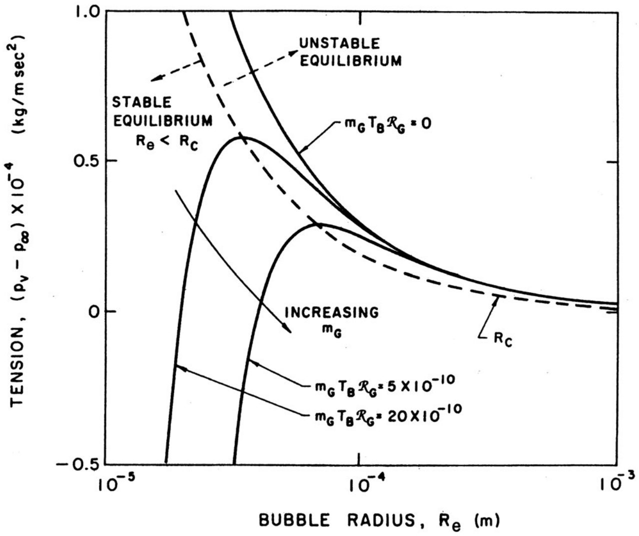

Figure 1: Stable and unstable bubble equilibrium radii as a function of the tension for various masses of gas in the bubble. Stable and unstable conditions are separated by the dotted line. Adapted from Daily and Johnson (1956).

This critical radius was first identified by Blake (1949) and Neppiras and Noltingk (1951) and is often referred to as the Blake critical radius. All bubbles of radius $R_e < R_c$ can exist in stable equilibrium, whereas all bubbles of radius $R_e > R_c$ must be unstable. This critical size could be reached by decreasing the ambient pressure from p_∞ to the critical value, $p_{\infty c}$, where from equations (Nge5) and (Nge1) it follows that

$$p_{\infty c} = p_V - \frac{4S}{3} \left\{ \frac{8\pi S}{9km_G T_B \mathcal{R}_G} \right\}^{\frac{1}{2}} \tag{Nge6}$$

which is often called the Blake threshold pressure.

The isothermal case ($k = 1$) is presented graphically in figure 1 where the solid lines represent equilibrium conditions for a bubble of size R_e plotted against the tension $(p_V - p_\infty)$ for various fixed masses of gas in the bubble and a fixed surface tension. The critical radius for any particular m_G corresponds to the maximum in each curve. The locus of the peaks is the graph of R_c values and is shown by the dashed line whose equation is $(p_V - p_\infty) = 4S/3R_e$. The region to the right of the dashed line represents unstable equilibrium conditions. This graphical representation was used by Daily and Johnson (1956) and is useful in visualizing the quasistatic response of a bubble when subjected to a decreasing pressure. Starting in the fourth quadrant under conditions in which the ambient pressure $p_\infty > p_V$, and assuming the mass of gas in the bubble is constant, the radius R_e will first increase as $(p_V - p_\infty)$ increases. The bubble will pass through a series of stable equilibrium states until the particular critical pressure corresponding to the maximum is reached. Any slight decrease in p_∞ below the value corresponding to this point will result in explosive cavitation growth regardless of whether p_∞ is further decreased or not. In the context of cavitation nucleation (Brennen 1995), it is recognized that a system consisting of small bubbles in a liquid can sustain a *tension* in the sense that it may be in equilibrium at liquid pressures below the vapor pressure. Due to surface tension, the maximum tension, $(p_V - p_\infty)$, that such a system could sustain would be $2S/R$. However, it is clear from the above analysis that stable equilibrium conditions do not exist in the range

$$\frac{4S}{3R} < (p_V - p_\infty) < \frac{2S}{R} \tag{Nge7}$$

and therefore the maximum tension should be given by $4S/3R$ rather than $2S/R$.