Pool Boiling Crisis

As a third and quite different example of the application of the drift flux method, we examine the two-phase flow associated with pool boiling, the background and notation for which were given in section (Nib). Our purpose here is to demonstrate the basic elements of two possible approaches to the prediction of boiling crisis. Specifically, we follow the approach taken by Zuber, Tribius and Westwater (1961) who demonstrated that the phenomenon of boiling crisis (the transition from nucleate boiling to film boiling) can be visualized as a flooding.

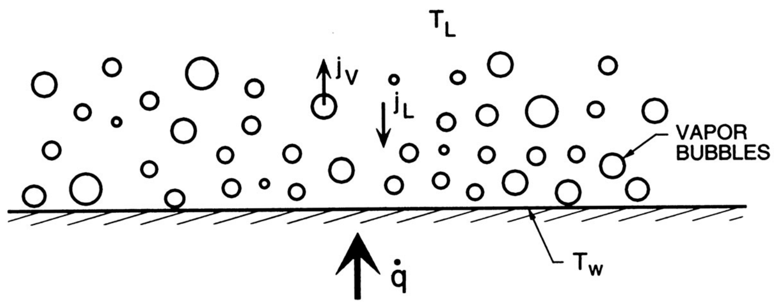

Figure 1: Nucleate boiling.

In the first analysis we consider the nucleate boiling process depicted in figure 1 and described in section (Nib). Using that information we can construct a drift flux chart for this flow as shown in figure 2.

It follows that, as illustrated in the figure, the operating point is given by the intersection of the drift flux curve, $j_{VL}(\alpha)$, with the dashed line

$$j_{VL} = \frac{\dot{q}}{\rho_V \mathcal{L}} \left\{ 1 - \alpha \left(1 - \frac{\rho_V}{\rho_L} \right) \right\} \approx \frac{\dot{q}}{\rho_V \mathcal{L}} (1 - \alpha) \tag{Nqe1}$$

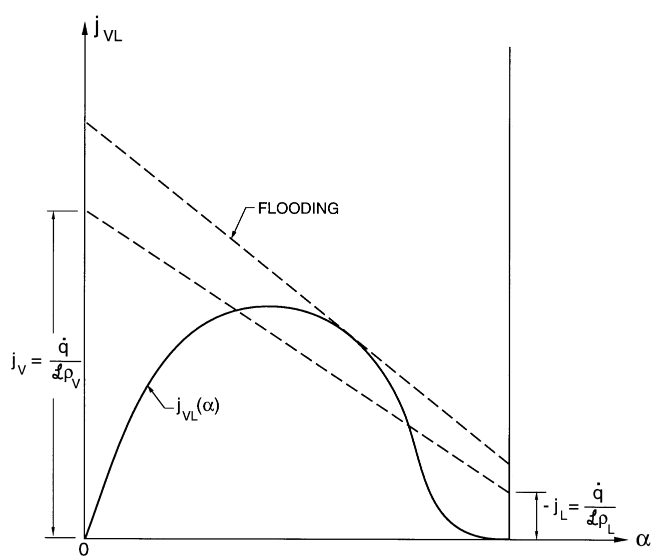

Figure 2: Drift flux chart for boiling.

where the second expression is accurate when $\rho_V/\rho_L \ll 1$ as is frequently the case. It also follows that this flow has a maximum heat flux given by the flooding condition sketched in figure 2. If the drift flux took the common form given by equation (Nqa2) and if $\rho_V/\rho_L \ll 1$ it follows that the maximum heat flux, $\dot{q}_{c1}$, is given simply by

$$\frac{\dot{q}_{c1}}{\rho_V \mathcal{L}} = K u_{VL0} \tag{Nqe2}$$

where, as before, u_{VL0}, is the terminal velocity of individual bubbles rising alone and K is a constant of order unity. Specifically,

$$K = \frac{1}{b}\left(1 - \frac{1}{b}\right)^{b-1} \tag{Nqe3}$$

so that, for $b = 2$, $K = 1/4$ and, for $b = 3$, $K = 4/27$.

It remains to determine u_{VL0} for which a prerequisite is knowledge of the typical radius of the bubbles, R. Several estimates of these characteristic quantities are possible. For purposes of an example, we shall assume that the radius is determined at the moment at which the bubble leaves the wall. If this occurs when the characteristic buoyancy force, $\frac{4}{3}\pi R^3 g(\rho_L - \rho_V)$, is balanced by the typical surface tension force, $2\pi SR$, then an appropriate estimate of the radius of the bubbles is

$$R = \left\{\frac{3S}{2g(\rho_L - \rho_V)}\right\}^{\frac{1}{2}} \tag{Nqe4}$$

Moreover, if the terminal velocity, u_{VL0}, is given by a balance between the same buoyancy force and a drag force given by $C_D \pi R^2 \rho_L u_{VL0}^2/2$ then an appropriate estimate of u_{VL0} is

$$u_{VL0} = \left\{\frac{8Rg(\rho_L - \rho_V)}{3\rho_L C_D}\right\}^{\frac{1}{2}} \tag{Nqe5}$$

Using these relations in the expression (Nqe2) for the critical heat flux, $\dot{q}_{c1}$, leads to

$$\dot{q}_{c1} = C_1 \rho_V \mathcal{L} \left\{\frac{Sg(\rho_L - \rho_V)}{\rho_L^2}\right\}^{\frac{1}{4}} \tag{Nqe6}$$

where C_1 is some constant of order unity. We shall delay comment on the relation of this maximum heat flux to the critical heat flux, $\dot{q}_c$, and on the specifics of the expression (Nqe6) until the second model calculation is completed.

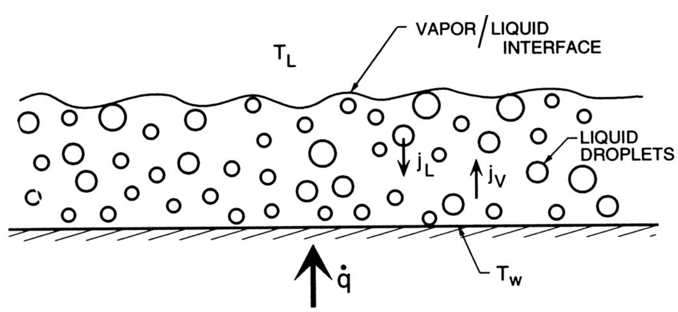

Figure 3: Sketch of the conditions close to film boiling.

A second approach to the problem would be to visualize that the flow near the wall is primarily within a vapor layer, but that droplets of water are formed at the vapor/liquid interface and drop through this vapor layer to impinge on the wall and therefore cool it (figure 3). Then, the flow within the vapor film consists of water droplets falling downward through an upward vapor flow rather than the previously envisaged vapor bubbles rising through a downward liquid flow. Up to and including equation (Nqe3), the analytical results for the two models are identical since no reference was made to the flow pattern. However, equations (Nqe4) and (Nqe5) must be re-evaluated for this second model. Zuber *et al.* (1961) visualized that the size of the water droplets formed at the vapor/liquid interface would be approximately equal to the most unstable wavelength, λ, associated with this Rayleigh-Taylor unstable surface (see section (Njo), equation (Njo2)) so that

$$R \approx \lambda \propto \left\{ \frac{S}{g(\rho_L - \rho_V)} \right\}^{\frac{1}{2}} \quad \text{(Nqe7)}$$

Note that, apart from a constant of order unity, this droplet size is functionally identical to the vapor bubble size given by equation (Nqe4). This is reassuring and suggests that both are measures of the *grain size* in this complicated, high void fraction flow. The next step is to evaluate the drift flux for this droplet flow or, more explicitly, the appropriate expression for u_{VL0}. Balancing the typical net gravitational force, $\frac{4}{3}\pi R^3 g(\rho_L - \rho_V)$ (identical to that of the previous bubbly flow), with a characteristic drag force given by $C_D \pi R^2 \rho_V u_{VL0}^2/2$ (which differs from the previous bubbly flow analysis only in that ρ_V has replaced ρ_L) leads to

$$u_{VL0} = \left\{ \frac{8Rg(\rho_L - \rho_V)}{3\rho_V C_D} \right\}^{\frac{1}{2}} \quad \text{(Nqe8)}$$

Then, substituting equations (Nqe7) and (Nqe8) into equation (Nqe2) leads to a critical heat flux, $\dot{q}_{c2}$, given by

$$\dot{q}_{c2} = C_2 \rho_V \mathcal{L} \left\{ \frac{Sg(\rho_L - \rho_V)}{\rho_V^2} \right\}^{\frac{1}{4}} \quad \text{(Nqe9)}$$

where C_2 is some constant of order unity.

The two model calculations presented above (and leading, respectively, to critical heat fluxes given by equations (Nqe6) and (Nqe9)) allow the following interpretation of the pool boiling crisis. The first model shows that the bubbly flow associated with nucleate boiling will reach a critical state at a heat flux given by $\dot{q}_{c1}$ at which the flow will tend to form a vapor film. However, this film is unstable and vapor droplets will continue to be detached and fall through the film to wet and cool the surface. As the heat flux is further increased a second critical heat flux given by $\dot{q}_{c2} = (\rho_L/\rho_V)^{\frac{1}{2}} \dot{q}_{c1}$ occurs beyond which it is no longer possible for the water droplets to reach the surface. Thus, this second value, $\dot{q}_{c2}$, will more closely predict the true boiling crisis limit. Then, the analysis leads to a dimensionless critical heat flux, $(\dot{q}_c)_{nd}$, from equation (Nqe9) given by

$$(\dot{q}_c)_{nd} = \frac{\dot{q}_c}{\rho_V \mathcal{L}} \left\{ \frac{Sg(\rho_L - \rho_V)}{\rho_V^2} \right\}^{-\frac{1}{4}} = C_2 \quad \text{(Nqe10)}$$

Kutateladze (1948) had earlier developed a similar expression using dimensional analysis and experimental data; Zuber *et al.* (1961) placed it on a firm analytical foundation.

Borishanski (1956), Kutateladze (1952), Zuber *et al.* (1961) and others have examined the experimental data on critical heat flux in order to determine the value of $(\dot{q}_c)_{nd}$ (or C_2) that best fits the data. Zuber *et al.* (1961) estimate that value to be in the range $0.12 \rightarrow 0.15$ though Rohsenow and Hartnett (1973) judge that 0.18 agrees well with most data. Figure 4 shows that the values from a wide range of experiments with fluids including water, benzene, ethanol, pentane, heptane and propane all lie within the $0.10 \rightarrow 0.20$. In that figure $(\dot{q}_C)_{nd}$ (or C_2) is presented as a function of the Haberman-Morton number, $Hm = g\mu_L^4(1 - \rho_V/\rho_L)/\rho_L S^3$, since, as was seen in section (Nfb), the appropriate type and size of bubble that is likely to form in a given liquid will be governed by Hm.

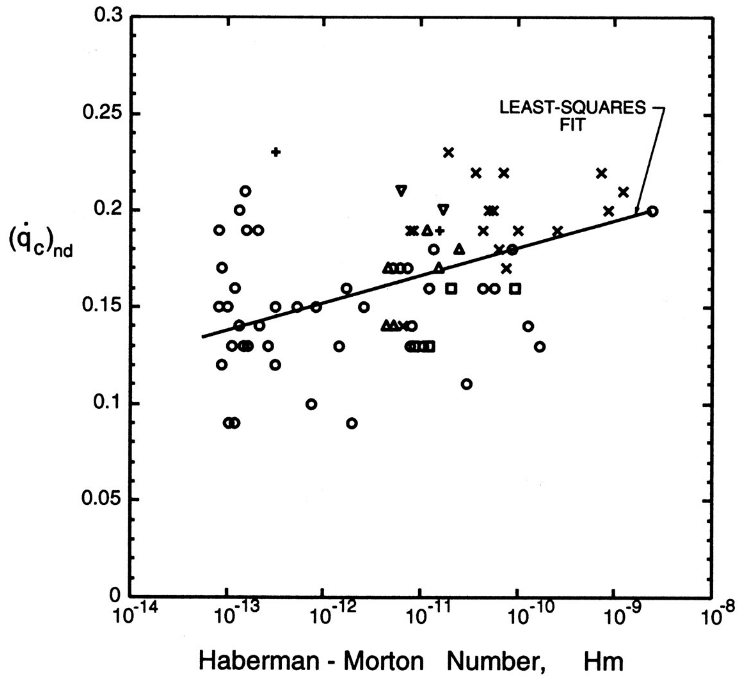

Figure 4: Data on the dimensionless critical heat flux, $(\dot{q}_c)_{nd}$ (or C_2), plotted against the Haberman-Morton number, $Hm = g\mu_L^4(1-\rho_V/\rho_L)/\rho_L S^3$, for water (+), pentane (×), ethanol (⊡), benzene (△), heptane(▽) and propane (∗) at various pressures and temperatures. Adapted from Borishanski (1956) and Zuber *et al.* (1961).

Lienhard and Sun (1970) showed that the correlation could be extended from a simple horizontal plate to more complex geometries such as heated horizontal tubes. However, if the typical dimension of that geometry (say the tube diameter, d) is smaller than λ (equation (Nqe7)) then that dimension should replace λ in the above analysis. Clearly this leads to an alternative correlation in which $(\dot{q}_c)_{nd}$ is a function of d; explicitly Lienhard and Sun recommend

$$(\dot{q}_c)_{nd} = 0.061/K^* \quad \text{where} \quad K^* = d/\left\{\frac{S}{g(\rho_L-\rho_V)}\right\}^{\frac{1}{2}} \tag{Nqe11}$$

(the constant, 0.061, was determined from experimental data) and that the result (Nqe11) should be employed when $K^* < 2.3$. For very small values of K^* (less than 0.24) there is no nucleate boiling regime and film boiling occurs as soon as boiling starts.

For useful reviews of the extensive literature on the critical heat flux in boiling, the reader is referred to Rohsenow and Hartnet (1973), Collier and Thome (1994), Hsu and Graham (1976) and Whalley (1987).