Disperse Phase Momentum Equation

At this point we should consider the relation between the equation of motion for an individual particle of the disperse phase and the Disperse Phase Momentum Equation (DPME) delineated in the last section. This relation is analogous to that between the number continuity equation and the Disperse Phase Continuity Equation (DPCE). The construction of the equation of motion for an individual particle in an infinite fluid medium will be discussed at some length in sections that follow. It is sufficient at this point to recognize that we may write Newton's equation of motion for an individual particle of volume v_D in the form

$$\frac{D_D}{D_D t}\left(\rho_D v_D u_{Dk}\right) = F_k + \rho_D v_D g_k \qquad \text{(Nbf48)}$$

where $D_D/D_D t$ is the Lagrangian time derivative following the particle so that

$$\frac{D_D}{D_D t} \equiv \frac{\partial}{\partial t} + u_{Di}\frac{\partial}{\partial x_i} \qquad \text{(Nbf2)}$$

and F_k is the force that the surrounding continuous phase imparts to the particle in the direction k. Note that F_k will include not only the force due to the velocity and acceleration of the particle relative to the fluid but also the *buoyancy* forces due to pressure gradients within the continuous phase. Expanding (Nbf1) and using the expression (Nbc4) for the mass interaction, $\mathcal{I}_D$, one obtains the following form of the DPME:

$$\rho_D v_D \left\{\frac{\partial u_{Dk}}{\partial t} + u_{Di}\frac{\partial u_{Dk}}{\partial x_i}\right\} + u_{Dk}\frac{\mathcal{I}_D}{n_D} = F_k + \rho_D v_D g_k \qquad \text{(Nbf3)}$$

Now examine the implication of this relation when considered alongside the IPME (Nbe8) for the disperse phase. Setting $\alpha_D = n_D v_D$ in equation (Nbe8), expanding and comparing the result with equation (Nbf3) (using the continuity equation (Nbb2)) one observes that

$$\mathcal{F}_{Dk} = n_D F_k \qquad \text{(Nbf4)}$$

Hence the appropriate force interaction term in the disperse phase momentum equation is simply the sum of the fluid forces acting on the individual particles in a unit volume, namely $n_D F_k$. As an example note that the steady, uniform sedimentation interaction force $\mathcal{F}_{Dk}$ given by equation (Nbe11), when substituted into equation (Nbf4), leads to the result $F_k = -\rho_D v_D g_k$ or, in words, a fluid force on an individual particle that precisely balances the weight of the particle.