# Basic Equations

In this chapter it is assumed that the ratio of liquid to vapor density is sufficiently large so that the volume of liquid evaporated or condensed is negligible. It is also assumed that bubbles are neither created or destroyed. Then the appropriate continuity equation is

$$\frac{\partial u_i}{\partial x_i} = \frac{\eta}{(1+\eta v)}\,\frac{Dv}{Dt} \tag{Nmb1}$$

where $\eta$ is the population or number of bubbles per unit volume of liquid and $v(x_i,t)$ is the volume of individual bubbles. The above form of the continuity equation assumes that $\eta$ is uniform; such would be the case if the flow originated from a uniform stream of uniform population and if there were no relative motion between the bubbles and the liquid. Note also that $\alpha = \eta v/(1+\eta v)$ and the mixture density, $\rho \approx \rho_L(1-\alpha) = \rho_L/(1+\eta v)$. This last relation can be used to write the momentum equation (Nla2) in terms of $v$ rather than $\rho$:

$$\rho_L \frac{Du_i}{Dt} = -(1+\eta v)\frac{\partial p}{\partial x_i} \tag{Nmb2}$$

The hydrostatic pressure gradient due to gravity has been omitted for simplicity.

Finally the Rayleigh-Plesset equation (Ngd2) relates the pressure $p$ and the bubble volume, $v = \frac{4}{3}\pi R^3$:

$$R\frac{D^2R}{Dt^2} + \frac{3}{2}\left(\frac{DR}{Dt}\right)^2 = \frac{p_V - p}{\rho_L} + \frac{p_{Go}}{\rho_L}\left(\frac{R_o}{R}\right)^{3k} - \frac{2S}{\rho_L R} - \frac{4\nu_L}{R}\frac{DR}{Dt} \tag{Nmb3}$$

where it is assumed that the mass of gas in the bubble remains constant, $p_V$ is the vapor pressure, $p_{Go}$ is the partial pressure of non-condensable gas at some reference moment in time when $R = R_o$ and $k$ is the polytropic index representing the behavior of the gas.

Equations (Nmb1), (Nmb2) and (Nmb3) can, in theory, be solved to find the unknowns $p(x_i,t)$, $u_i(x_i,t)$, and $v(x_i,t)$ (or $R(x_i,t)$) for any bubbly cavitating flow. In practice the nonlinearities in the Rayleigh-Plesset equation and in the Lagrangian derivative, $D/Dt = \partial/\partial t + u_i\partial/\partial x_i$, present serious difficulties for all flows except those of the simplest geometry. In the following sections several such flows are examined in order to illustrate the interactive effects of bubbles in cavitating flows and the role played by bubble dynamics in homogeneous flows.