Normal Shock Waves in Bubbly Flows

The propagation and structure of shock waves in bubbly cavitating flows represent a rare circumstance in which fully nonlinear solutions of the governing equations can be obtained. Shock wave analyses of this kind were investigated by Campbell and Pitcher (1958), Crespo (1969), Noordzij (1973), and Noordzij and van Wijngaarden (1974), among others, and for more detail the reader should consult these works. Since this chapter is confined to flows without significant relative motion, this section will not cover some of the important effects of relative motion on the structural evolution of shocks in bubbly liquids. For this the reader is referred to Noordzij and van Wijngaarden (1974).

Consider a normal shock wave in a coordinate system moving with the shock so that the flow is steady and the shock stationary (figure 1). If x and u represent a coordinate and the fluid velocity normal to the shock, then continuity requires

$$\rho u = \text{constant} = \rho_1 u_1 \tag{Nme1}$$

where ρ_1 and u_1 will refer to the mixture density and velocity far upstream of the shock. Hence u_1 is also the velocity of propagation of a shock into a mixture with conditions identical to those upstream of the shock. It is assumed that $\rho_1 \approx \rho_L(1-\alpha_1) = \rho_L/(1+\eta v_1)$ where the liquid density is considered constant and α_1, $v_1 = \frac{4}{3}\pi R_1^3$, and η are the void fraction, individual bubble volume, and population of the mixture far upstream.

Substituting for ρ in the equation of motion and integrating, one also obtains

$$p + \frac{\rho_1^2 u_1^2}{\rho} = \text{constant} = p_1 + \rho_1 u_1^2 \tag{Nme2}$$

This expression for the pressure, p, may be substituted into the Rayleigh-Plesset equation using the observation that, for this steady flow,

$$\frac{DR}{Dt} = u\frac{dR}{dx} = u_1 \frac{(1+\eta v)}{(1+\eta v_1)}\ \frac{dR}{dx} \tag{Nme3}$$

$$\frac{D^2R}{Dt^2} = u_1^2 \frac{(1+\eta v)}{(1+\eta v_1)^2}\left[(1+\eta v)\frac{d^2R}{dx^2} + 4\pi R^2 \eta \left(\frac{dR}{dx}\right)^2\right] \tag{Nme4}$$

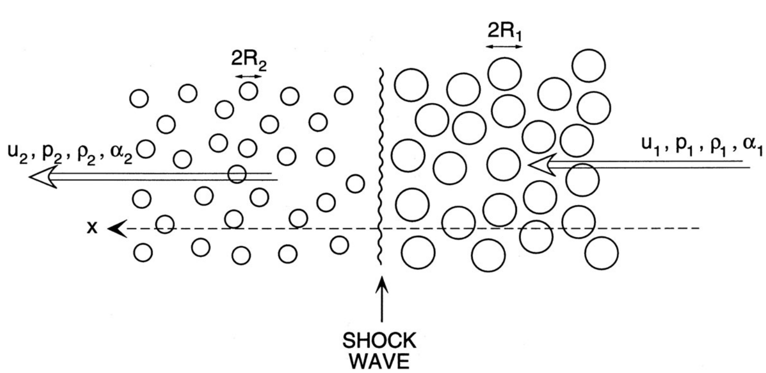

Figure 1: Schematic of the flow relative to a bubbly shock wave.

where $v = \frac{4}{3}\pi R^3$ has been used for clarity. It follows that the structure of the flow is determined by solving the following equation for $R(x)$:

$$u_1^2 \frac{(1+\eta v)^2}{(1+\eta v_1)^2} R \frac{d^2 R}{dx^2} + \frac{3}{2} u_1^2 \frac{(1+3\eta v)(1+\eta v)}{(1+\eta v_1)^2} \left(\frac{dR}{dx} \right)^2 \tag{Nme5}$$

$$+ \frac{2S}{\rho_L R} + \frac{u_1(1+\eta v)}{(1+\eta v_1)} \frac{4\nu_L}{R} \left(\frac{dR}{dx} \right) = \frac{(p_B - p_1)}{\rho_L} + \frac{\eta(v - v_1)}{(1+\eta v_1)^2} u_1^2$$

It will be found that dissipation effects in the bubble dynamics strongly influence the structure of the shock. Only one dissipative effect, namely that due to viscous effects (last term on the left-hand side) has been explicitly included in equation (Nme5). However, as discussed in the last section, other dissipative effects may be incorporated approximately by regarding ν_L as a total *effective* viscosity.

The pressure within the bubble is given by

$$p_B = p_V + p_{G1} \left(v_1/v \right)^k \tag{Nme6}$$

and the equilibrium state far upstream must satisfy

$$p_V - p_1 + p_{G1} = 2S/R_1 \tag{Nme7}$$

Furthermore, if there exists an equilibrium state far downstream of the shock (this existence will be explored shortly), then it follows from equations (Nme5) and (Nme6) that the velocity, u_1, must be related to the ratio, R_2/R_1 (where R_2 is the bubble size downstream of the shock), by

$$u_1^2 = \frac{(1-\alpha_2)}{(1-\alpha_1)(\alpha_1 - \alpha_2)} \left[\frac{(p_1 - p_V)}{\rho_L} \left\{ \left(\frac{R_1}{R_2} \right)^{3k} - 1 \right\} + \frac{2S}{\rho_L R_1} \left\{ \left(\frac{R_1}{R_2} \right)^{3k} - \frac{R_1}{R_2} \right\} \right] \tag{Nme8}$$

where α_2 is the void fraction far downstream of the shock and

$$\left(\frac{R_2}{R_1} \right)^3 = \frac{\alpha_2(1-\alpha_1)}{\alpha_1(1-\alpha_2)} \tag{Nme9}$$

Hence the *shock velocity,* u_1, is given by the upstream flow parameters α_1, $(p_1 - p_V)/\rho_L$, and $2S/\rho_L R_1$, the polytropic index, k, and the downstream void fraction, α_2. An example of the dependence of u_1 on α_1 and α_2 is shown in figure 2 for selected values of $(p_1 - p_V)/\rho_L = 100\ m^2/sec^2$, $2S/\rho_L R_1 = 0.1\ m^2/sec^2$, and $k = 1.4$. Also displayed by the dotted line in this figure is the sonic velocity of the mixture (at zero frequency), c_1, under the upstream conditions; it is readily shown that c_1 is given by

$$c_1^2 = \frac{1}{\alpha_1(1-\alpha_1)} \left[\frac{k(p_1 - p_V)}{\rho_L} + \left(k - \frac{1}{3} \right) \frac{2S}{\rho_L R_1} \right] \tag{Nme10}$$

Alternatively, the presentation conventional in gas dynamics can be adopted. Then the upstream Mach number, u_1/c_1, is plotted as a function of α_1 and α_2. The resulting graphs are functions only of two parameters, the polytropic index, k, and the parameter, $R_1(p_1 - p_V)/S$. An example is included as figure 3 in which $k = 1.4$ and $R_1(p_1 - p_V)/S = 200$. It should be noted that a real shock velocity and a real sonic speed can exist even when the upstream mixture is under tension ($p_1 < p_V$). However, the numerical value of the tension, $p_V - p_1$, for which the values are real is limited to values of the parameter $R_1(p_1 - p_V)/2S > -(1 - 1/3k)$ or -0.762 for $k = 1.4$. Also note that figure 3 does not change much with the parameter, $R_1(p_1 - p_V)/S$.

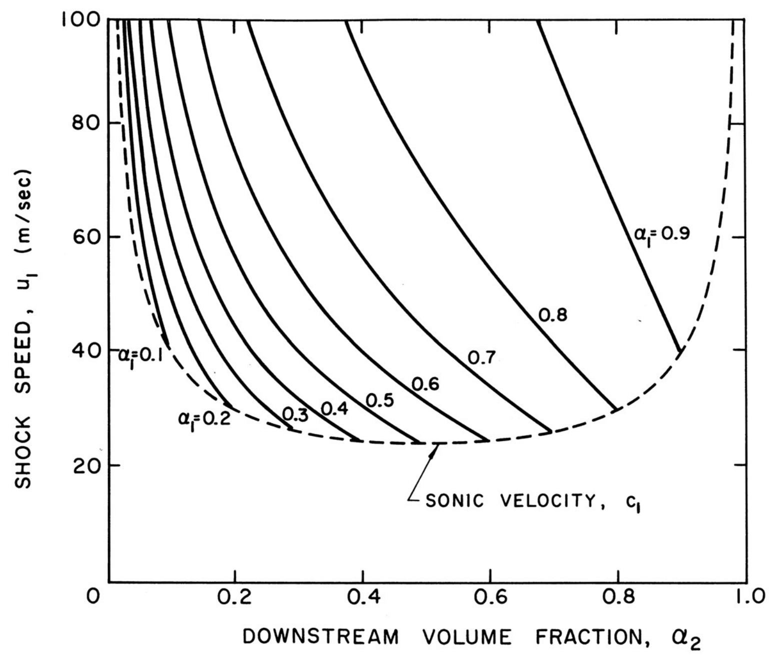

Figure 2: Shock speed, u_1, as a function of the upstream and downstream void fractions, α_1 and α_2, for the particular case $(p_1 - p_V)/\rho_L = 100\ m^2/sec^2$, $2S/\rho_L R_1 = 0.1\ m^2/sec^2$, and $k = 1.4$. Also shown by the dotted line is the sonic velocity, c_1, under the same upstream conditions.

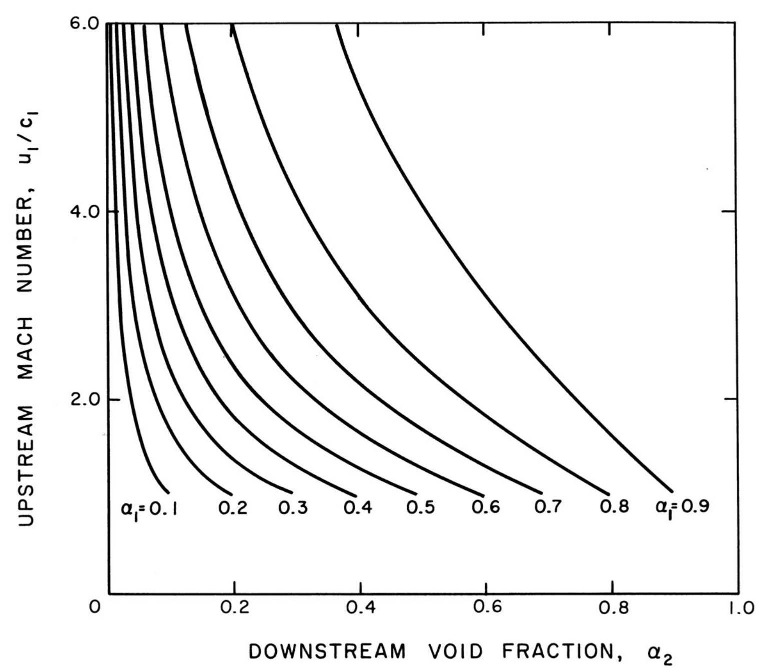

Figure 3: The upstream Mach number, u_1/c_1, as a function of the upstream and downstream void fractions, α_1 and α_2, for $k = 1.4$ and $R_1(p_1 - p_V)/S = 200$.