Kinematic Wave Speed at Flooding

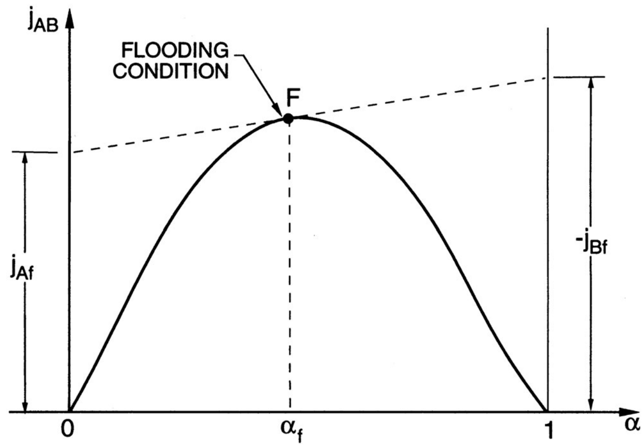

Figure 1: Conditions of flooding at a volume fraction of α_f and volume fluxes j_{Af} and j_{Bf}.

In section (Nqc) we identified the phenomenon of flooding and drew the analogies to choking in gas dynamics and open-channel flow. Note that in these analogies, the choked flow is independent of conditions downstream because signals (small amplitude waves) cannot travel upstream through the choked flow since the fluid velocity there is equal to the small amplitude wave propagation speed relative to the fluid. Hence in the laboratory frame, the upstream traveling wave speed is zero. The same holds true in a flooded flow as illustrated in figure 1 which depicts flooding at a volume fraction of α_f and volume fluxes, j_{Af} and j_{Bf}. From the geometry of this figure it follows that

$$j_{Af} + j_{Bf} \quad = \quad j_f \quad = \quad = - \left. \frac{dj_{AB}}{d\alpha} \right|_{\alpha_f} \qquad \text{(Nsc1)}$$

and therefore the kinematic wave speed at the flooding condition, c_f is

$$c_f \quad = \quad j_f + \left. \frac{dj_{AB}}{d\alpha} \right|_{\alpha_f} \quad = \quad 0 \qquad \text{(Nsc2)}$$

Thus the kinematic wave speed in the laboratory frame is zero and small disturbances cannot propagate through flooded flow. Consequently, the flow is choked just as it is in the gas dynamic or open channel flow analogies.

One way to visualize this limit in a practical flow is to consider countercurrent flow in a vertical pipe whose cross-sectional area decreases as a function of axial position until it reaches a throat. Neglecting the volume fraction changes that could result from the changes in velocity and therefore pressure, the volume flux intercepts in figure 1, j_A and j_B, therefore increase with decreasing area. Flooding or choking will occur at a throat when the fluxes reach the flooding values, j_{Af} and j_{Bf}. The kinematic wave speed at the throat is then zero.