Averaging in Pipe Flows

One specific example of a quadratic rectification term (in this case a discrepancy between the product of an average and the average of a product) is that recognized by Zuber and Findlay (1965). In order to account for the variations in velocity and volume fraction over the cross-section of a pipe in constructing the one-dimensional equations of pipe flow, they found it necessary to introduce a distribution parameter, C_0, defined by

$$C_0 = \frac{\overline{\alpha j}}{\overline{\alpha}\ \ \overline{j}} \tag{Ndd1}$$

where the overbar now represents an average over the cross-section of the pipe. The importance of C_0 is best demonstrated by observing that it follows from equations (Nac16) that the cross-sectionally averaged volume fraction, $\overline{\alpha_A}$, is now related to the volume fluxes, $\overline{j_A}$ and $\overline{j_B}$, by

$$\overline{\alpha_A} = \frac{1}{C_0} \frac{\overline{j_A}}{(\overline{j_A} + \overline{j_B})} \tag{Ndd2}$$

Values of C_0 of the order of 1.13 (Zuber and Findlay 1965) or 1.25 (Wallis 1969) appear necessary to match the experimental observations.